One method for efficiently embedding Hamiltonian cycle in Möbius cubes

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Abstract

The Möbius cube MQ_n is a variant of the hypercube structure that has better performance with the same number of links and processors. The cycle is a popular interconnection topology and has been widely used in distributed-memory parallel computers. Moreover, the parallel algorithms of cycles have been extensively developed. In this paper, we propose one efficient method to generate Hamiltonian cycle in Möbius cubes and give a conjecture that some permutations are not available to generate cycle.

1 Introduction

Interconnection networks play a major role in the performance of distributed-memory multiprocessor and the one primary concern for choosing an appropriate interconnection network is the graph embedding ability. The graph embedding is the mapping of a topological structure (guest graph) into other topological structure (host graph) that preserves certain required topological properties and the graph embedding ability reflects how efficiently a parallel algorithm with guest graph can be executed on host graph and the utilization of system resources in host graph. Many applications, such as architecture simulations and processor allocations, can be modeled as graph embedding.

A cycle structure is a fundamental network for multiprocessor systems and suitable for developing simple algorithms with low communication costs. Several efficient algorithms have been designed with respect to cycle-structures for solving a variety of algebraic problems, graph problems, and some parallel applications, such as those in image and signal processing. Due to efficiently executing a parallel program, the targeted interconnection network possesses a Hamiltonian cycle, i.e., a cycle passing every vertex of the network exactly once if the number of processes in the ring-structure parallel algorithm equals the number of vertices of the interconnection network.

With regard to the cycles embedding of interconnection networks, many interesting results have received much attention [1, 3, 12, 13]. In particular, Zheng and Latifi [13] introduced the notion of the reflected link label sequences and proposed a kind of codeword, termed the Generalized Gray Code. In this paper, we consider the problem of embedding a Hamiltonian cycle in the Möbius cube. We adopt concepts of reflected link label sequences and cycle pattern in [3, 13] and use them to construct an efficient algorithm for embedding a desired Hamiltonian cycle in the Möbius cube.

An n-dimensional Möbius cube, MQ_n, is an important alternative of hypercube Q_n. MQ_n has 2^n nodes and $n2^{n-1}$ edges. The diameter of MQ_n is about one half that of the n-dimensional hypercube Q_n and the average number of steps between nodes for MQ_n is about two-thirds of the average for Q_n, and $1 - MQ_n$ has dynamic performance superior to that of Q_n. Of course, the symmetry of MQ_n is not superior to that of Q_n, i.e., Q_n is both node symmetric and edge symmetric, whereas MQ_n is, in general, neither node symmetric ($n \geq 4$) nor edge symmetric ($n \geq 3$). In recent years, there are many research on the Möbius cubes. [4, 5, 6, 7, 8]

In particular, J. Fan [4] proved that any cycle of length l, $4 \leq l \leq 2^n$, can be embedded into MQ_n with dilation 1 ($n \geq 2$) by using the

Hamilton-connectivity of MQ_n. However, we propose one systematic way to generate cycles with concepts of reflected link label sequences and cycle pattern in this paper.

The rest of this paper is organized as follows. The preliminary knowledge and fundamental definitions are given in next section. Then some interesting permutations are introduced for constructing desired Hamiltonian cycles in Section 3. Conclusions are given in the final section.

2 Preliminaries

We give here the basic graph-theoretic definitions relevant to this paper. The topology of an interconnection network is conveniently represented by an undirected simple graph $G = (V, E)$, where $V(G)$ and $E(G)$ are the vertex set and the edge set of G, respectively. Throughout this paper, the terms graph and network are used interchangeably. Moreover, the terms node and vertex are used interchangeably. For graph terminology and notation not defined here we refer the reader to [9, 11].

A *walk* in a graph is a finite sequence $\omega : \langle \lambda_0, e_1, \lambda_1, e_2, \lambda_2, e_3 \ldots, \lambda_{l-1}, e_l, \lambda_l \rangle$ whose terms are alternately vertices and edges so, for $1 \le i \le l$, the edge e_i has ends λ_{i-1} and λ_i, thus each edge e_i is immediately preceded and succeeded by the two vertices with which it is incident. A *path* is a sequence of adjacent vertices written as $\langle u_0, u_1, \ldots, u_l \rangle$, and all the vertices are distinct. A *cycle*, $\langle u_0, u_1, \ldots, u_l \rangle$, is a path which at least three vertices and $u_0 = u_l$. For a cycle which traversing all the vertices on a graph, we call it as $Hamiltonian\ cycle$.

An n-dimensional Möbius cube has 2^n nodes. Each node has a label with unique n-bits binary code and n neighbors. For instance, a node x has a label $x_{n-1}x_{n-2}\ldots x_0$. A node y is called i-neighbor of x if and only if $y = x_{n-1}x_{n-2}\ldots x_{i+1}\overline{x_i}x_{i-1}x_{i-2}\ldots x_0$ if $x_{i+1} = 0$ or $y = x_{n-1}x_{n-2}\ldots x_{i+1}\overline{x_i}\overline{x_{i-1}}\ldots \overline{x_1}\overline{x_0}$ if $x_{i+1} = 1$.

More informally, x connects to a neighbor that differs in bit x_i if $x_{i+1} = 0$, and to a neighbor that differs in bits x_i through x_0, if $x_{i+1} = 1$. The connection between x and y along dimension $n-1$ has x_n undefined, so we can assume x_n is either equal to 0 or equal to 1, which gives us slightly different network topologies. If we assume $x_n = 0$, we call the network a 0-Möbius cube; and if we assume $x_n = 1$, we call the network a 1-Möbius cube.

An 1-dimensional Möbius cube is a graph with two connected nodes. An n-dimensional Möbius cube is formed with an $(n-1)$-dimensional 0-Möbius cube and an $(n-1)$-dimensional 1-Möbius cube. From the definition of the Möbius cube, we can construct an n-dimensional Möbius cube by adding 2^{n-1} edges. For convenience, we will write MQ_n^0 and MQ_n^1 as n-dimensional 0-Möbius cube and n-dimensional 1-Möbius cube, respectively.

Figs. 1 and 2 show the MQ_4^0 and MQ_4^1. They also demonstrate the expansibility of the Möbius cube networks by showing how a 0-Möbius cube of dimension 3 connects to a 1-Möbius cube of dimension 3 to create a 0-Möbius cube or a 1-Möbius cube of dimension 4.

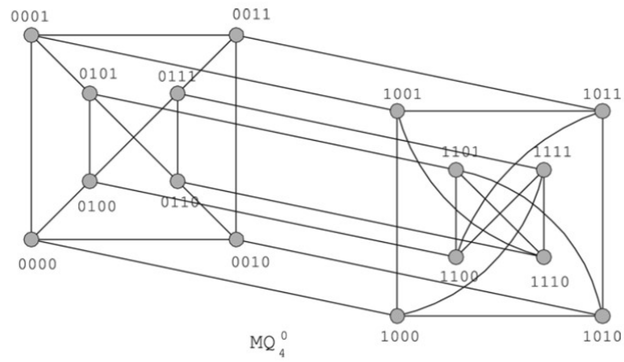

Figure 1: 4-dimensional 0-Möbius cube

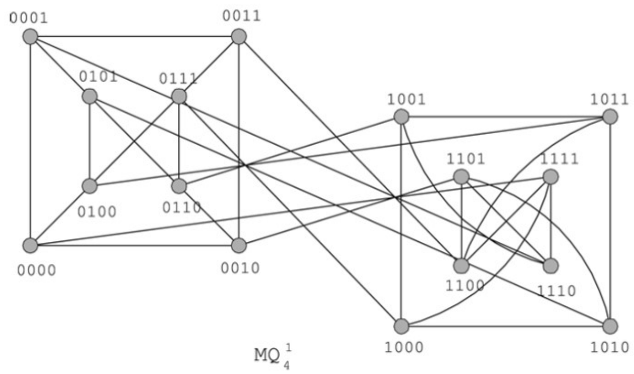

Figure 2: 4-dimensional 1-Möbius cube

3 Permutation and edge label sequence

We say that v is the i-neighbor of u, denoted by $v = u^i$, and the edge (u, v) is an edge of dimension i if the following two conditions are satisfied: 1) $u_{n-1}u_{n-2}\dots u_iu_{i-1}\dots u_1u_0 = u_{n-1}u_{n-2}\dots u_{i+1}\overline{u}_iu_{i-1}\dots u_1u_0$ for $u_{i+1} = 0$; and 2) $u_{n-1}u_{n-2}\dots u_iu_{i-1}\dots u_1u_0 = u_{n-1}u_{n-2}\dots u_{i+1}\overline{u}_i\overline{u}_{i-1}\dots \overline{u}_1\overline{u}_0$ for $u_{i+1} = 1$. For instance, for vertex $u = 10010$ of MQ_5^0, its 4-, 3-, 2-, 1-, and 0-neighbors are 00010, 11101, 10110, 10000, 10011, respectively. Let $v = u^{ij}$ denote the j-neighbor of u^i.

To convenience, we will usually use a vertex and an edge label sequence to identify a desired path or cycle throughout this paper. Some graph-theoretic notations and terminology for our purpose are given as follows. Let $L(n) = \{0, 1, \dots, n-1\}$ denote the set consisting of n dimensions in a MQ_n. An *edge label sequence* S is a sequence generated by the elements of $L(n)$. To simplify the explanation, if no ambiguity arises, the terms S and $[S]$ are used interchangeably for an edge label sequence in the following discussions. For instance, [12] and [020] are two edge label sequences in a MQ_3. We can use an edge label sequence and start/last vertices to represent a path in a MQ_n explicitly. For instance, we write 000[12]110 and 000[020]100 to denote the two paths $\langle 000, 010, 110\rangle$ and $\langle 000, 001, 101, 100\rangle$, respectively, in a MQ_3^0.

A walk, $\omega(S, v) = \langle \lambda_0, \lambda_1, \lambda_2, \dots, \lambda_m\rangle$, in a MQ_n can be generated with respect to a given edge label sequence $S = d_1d_2\cdots d_m$ and a given vertex v as follows: $\lambda_0 = v$, and λ_j is the d_j-neighbor of λ_{j-1} in a MQ_n where $1 \le j \le m$, that is, $\lambda_{j-1}[d_j]\lambda_j$ is an edge with label d_j. Thus, this walk $\omega(S, v)$ is also represented as $\langle\lambda_0[S]\lambda_m\rangle$ or $\lambda_0[S]\lambda_m$. In particular, the edge label sequence S is interesting when it generates a loop-free path $\omega(S, v)$ starting from any vertex v in a MQ_n. A sequence S called *cycle sequence* if and only if $\omega(S, v)$ is a cycle.

Let a *proper subsequence* S_1 of sequence S be a nonempty subsequence with $S_1 \neq S$. Naturally, for a path $u[S]u'$ and a proper subsequence S_1 of S, $v \neq u$ and $v \neq u'$ for any vertex $v = u^{S_1}$. For comprehending the difference of start/last vertices u and v, let $DB(u, v)$ denote the set containing these different bit positions between them. Clearly, $DB(u, u^{ij}) = \{i, j\}$ if u_{i+1}, u_{j+1} are both 0. Clearly, for $v = u^S$, $DB(u, v)$ is dependent on elements of S and u. However, we prefer the edge label sequence S that $DB(u, v)$ is not dependent on u but S. For such edge label sequence S, we call it an *independent edge label sequence* (or IE sequence for short) and furthermore define $DB(S) = DB(u, v)$ if S is an IE sequence.

Let $D(s) = (d_0, d_1 \dots, d_{s-1})$ be a permutation of s, $1 \le s \le n$, elements taken from $L(n)$. We now define some useful concepts about permutation and reflected edge label sequence corresponding to $D(s)$.

Definition 1. *[10] For $n \ge 3$ and $1 \le s \le n$, a reflected edge label sequence $R_{D(s)}$ corresponding to a permutation $D(s) = (d_0, d_1, \dots, d_{s-1})$ is defined as:*

$$D(1) = d_0,\ R_{D(1)} = d_0,$$

$$D(k) = D(k-1) \cup d_{k-1},$$

$$R_{D(k)} = R_{D(k-1)} + d_{k-1} + R_{D(k-1)},\ 2 \le k \le s.$$

A *complete reflected edge label* (or complete for short) *sequence* corresponding to $D(s)$ is defined as $C_{D(s)} = R_{D(s)} + d_{s-1}$

A permutation $D(s)$ is called a *cycle permutation* if its complete sequence $C_{D(s)}$ is a cycle sequence. For instance, for $D(2) = \{0, 2\}$, $R_{D(2)}(2) = 020$, $C_{D(2)} = 0202$, and set $U = \{0, 2, 02, 20, 020, 202\}$ contains all proper subsequences of 0202. It is not difficult to verify that for any vertex $u \in MQ_3$, $DB(u, u^{0202}) = DB(0202) = DB(0022) = \varnothing$ and $DB(u, u^{S_1}) = DB(S_1) \neq \varnothing$ for each $S_1 \in U$. Thus, $C_{D(2)}$ is a cycle sequence and then $D(2)$ is a cycle permutation in MQ_3. In this paper, we provide one kind of cycle permutations to embed Hamiltonian cycles into a MQ_n.

Next we give some properties of complete reflected edge label sequence which is useful for us to prove our main result.

By Definition 1, we have Property 1.

Property 1. *Let $D(s) = (d_0, d_1 \dots, d_{s-1})$. There are 2^s elements in the sequence $C_{D(s)}$.*

Property 2. *Let $D(s) = (d_0, d_1 \ldots, d_{s-1})$. There are 2^{i-j} d_j's and 2^0 d_{i+1} among two conjunctive d_i's for $1 \leq i < s-1$, $0 \leq j < i$ in the sequence $C_{D(s)}$.*

Property 3. *Let $D(s) = (d_0, d_1 \ldots, d_{s-1})$. There are 2^{s-2-j} d_j's among two conjunctive d_{s-1}'s for $0 \leq j < s-1$ and a d_j among two conjunctive d_0's for $1 \leq j \leq s-1$ in the sequence $C_{D(s)}$.*

For instance, let $D(3) = (d_0, d_1, d_2)$ and $C_{D(3)} = d_0 d_1 d_0 d_2 d_0 d_1 d_0 d_2$. For Property 2, suppose that $i = 1$ and $j = 0$. Clearly, among two d_1's, there are $2^{1-0} = 2$ d_0's and $2^0 = 1$ d_2. And for Property 3, there are $2^{2-1-1} = 1$ d_1 and $2^1 = 2$ d_0's. For $i = 0$, we know that there is only one d_j among two d_0's where $1 \leq j \leq 2$.

4 Cycle permutation on Möbius cubes

In this section, we give a method to generate a Hamiltonian cycle and also give some permutations which are not available to generate a cycle.

A permutation $D(s) = (d_0, d_1 \ldots, d_{s-1})$, called *ascending* if for any integer i, $d_i < d_{i+1}$, where $0 \leq i < s$. And a permutation $D(s) = (1, 2 \ldots, s-1)$, called *consecutive ascending* (CA for short) for $2 \leq s \leq n$. We will use $D_A(s)$ and $D_{CA}(s)$ to represent an ascending and a consecutive ascending permutation, respectively.

Lemma 1. *Let $D_{CA}(n) = (1, 2 \ldots, n-1)$. $R_{D_{CA}(n)}$ is an IE sequence, $DB(R_{D_{CA}(n)}) = \{n-1, n-3, n-4, \ldots, 1, 0\}$ for MQ_n^0 and $DB(R_{D_{CA}(n)}) = \{n-1, n-2\}$ for MQ_n^1 where $n \geq 3$.*

Proof. We prove this lemma by induction on n and now discuss the base case for $n = 3$. For $D_{CA}(3) = (1, 2)$ and $R_{D_{CA}(3)} = 121$, by Table 1 and 2, $DB(R_{D_{CA}(3)}) = \{2, 0\}$ for MQ_3^0 and $DB(R_{D_{CA}(3)}) = \{2, 1\}$ for MQ_3^1. That is, this lemma is true for $n = 3$.

Suppose that this lemma holds for $n = k$. For $n = k+1$, we have $D_{CA}(k+1) = (1, 2 \ldots, k)$ and $R_{D_{CA}(k+1)} = [R_{D_{CA}(k)}, k, R_{D_{CA}(k)}]$. Let $u \in V(MQ_{k+1}^0)$ and $v = u^{R_{D_{CA}(k)}}$. By the induction hypothesis, $DB(u, v) = \{k-1, k-3, ..., 1, 0\}$ (resp. $DB(u, v) = \{k-1, k-2\}$) if $u_k = 0$ (resp. if $u_k = 1$). Let $x = v^k$ and $y = x^{R_{D_{CA}(k)}}$. Then $DB(v, x) = \{k\}$ and $DB(x, y) = \{k-1, k-2\}$ (resp. $DB(x, y) = \{k-1, k-3, ...1, 0\}$) if $u_k = 0$ (resp. if $u_k = 1$). Note that $DB(u, u^{R_{D_{CA}(k+1)}}) = DB(u, y) = DB(u, v) \triangle DB(v, x) \triangle DB(x, y) = \{k-1, k-3, ..., 1, 0\} \triangle \{k\} \triangle \{k-1, k-2\} = \{k, k-2, k-3, ..., 1, 0\}$ (resp. $DB(u, u^{R_{D_{CA}(k+1)}}) = \{k-1, k-2\} \triangle \{k\} \triangle \{k-1, k-3, ...1, 0\} = \{k, k-2, k-3, ..., 1, 0\}$) if $u_k = 0$ (resp. if $u_k = 1$). Now let $u \in V(MQ_{k+1}^1)$, we can have $DB(u, u^{R_{D_{CA}(k+1)}}) = \{k, k-1\}$ easily by using the same way we used before. This completes the proof. ☐

In the following, we introduce some special permutations which can help us to generate a Hamiltonian cycle. A permutation $D(s) = (d_0, d_1 \ldots, d_{s-2}, d_{s-1})$ is called *zero end*. A permutation is called *consecutive ascending zero end* if the final element d_{s-1} is zero (CZ for short) if $D(s) = (d_0, d_1 \ldots, d_{s-2}, 0)$ where $d_{i+1} = d_i + 1$ for $0 \leq i < s-1$.

Lemma 2. *Let $D_{CZ}(n) = (1, 2 \ldots, n-1, 0)$. For any vertex $u \in MQ_n$, $DB(u, u^{C_{D_{CZ}(n)}}) = \varnothing$.*

Proof. We prove this lemma by induction on n and now discuss the base case for $n = 2$. We have $D_{CZ}(2) = (1, 0)$ and $C_{D_{CZ}(2)} = 1010$. For $u \in MQ_2^0$, $DB(u, u^1) = \{1\}$, $DB(u, u^{10}) = \{1, 0\}$, $DB(u, u^{101}) = \{0\}$, and $DB(u, u^{1010}) = \varnothing$. For $u \in MQ_2^1$, $DB(u, u^1) = \{1, 0\}$, $DB(u, u^{10}) = \{1\}$, $DB(u, u^{101}) = \{0\}$, and $DB(u, u^{1010}) = \varnothing$.

Suppose that this lemma holds for $n = k$. For $n = k+1$, we have $D_{CZ}(k+1) = (1, 2 \ldots, k, 0)$ and $C_{D_{CZ}(k+1)} = [R_{D_{CA}(k)}, 0, R_{D_{CA}(k)}, 0]$. Let $u \in V(MQ_{k+1}^0)$ and $v = u^{R_{D_{CA}(k)}}$, by Lemma 1, we have $DB(u, v) = \{k, k-2, k-3 \ldots, 1, 0\}$. Let $x = v^0$, $y = x^{R_{D_{CA}(k)}}$ and $z = y^0$. Then $DB(u, x) = \{k, k-2, k-3 \ldots, 1\}$, $DB(u, y) = \{0\}$, and $DB(u, z) = \varnothing$. And if $u \in V(MQ_{k+1}^1)$, we have $DB(u, v) = \{k, k-1\}$, $DB(u, x) = \{k, k-1, 0\}$, $DB(u, y) = \{0\}$, and $DB(u, z) = \varnothing$. The proof is complete.

☐

Theorem 1. *For any vertex $u \in V(MQ_n)$, $\omega(C_{D_{CZ}(n)}, u)$ is a Hamiltonian cycle.*

Proof. We prove this theorem by induction on n and now discuss the base case for $n = 2$. We have $D_{CZ}(2) = (1, 0)$ and $C_{D_{CZ}(2)} = 1010$. For any vertex $u \in V(MQ_n)$, by Table 3 and Table 4, we

Table 1: All paths generated by $R_{(1,2)}$ in MQ_3^0

u	1	2	1	$DB(R_{(1,2)})$
000	010	110	101	$\{2,0\}$
001	011	111	100	$\{2,0\}$
010	000	100	111	$\{2,0\}$
011	001	101	110	$\{2,0\}$
100	111	011	001	$\{2,0\}$
101	110	010	000	$\{2,0\}$
110	101	001	011	$\{2,0\}$
111	100	000	010	$\{2,0\}$

Table 2: All paths generated by $R_{(1,2)}$ in MQ_3^1

u	1	2	1	$DB(R_{(1,2)})$
000	010	101	110	$\{2,1\}$
001	011	100	111	$\{2,1\}$
010	000	111	100	$\{2,1\}$
011	001	110	101	$\{2,1\}$
100	111	000	010	$\{2,1\}$
101	110	001	011	$\{2,1\}$
110	101	010	000	$\{2,1\}$
111	100	011	001	$\{2,1\}$

Table 3: All cycles generated by $C_{D_{CZ}(2)}$ in MQ_n^0

u	0	1	0	1
00	01	11	10	00
01	00	10	11	01
10	11	01	00	10
11	10	00	01	11

Table 4: All cycles generated by $C_{D_{CZ}(2)}$ in MQ_n^1

u	0	1	0	1
00	01	10	11	00
01	00	11	10	01
10	11	00	01	10
11	10	01	00	11

know that this theorem is true for $n = 2$.

Suppose that this theorem is holds for $n = k$. For $n = k+1$, we have $D_{CZ}(k+1) = (1, 2 \ldots, k, 0)$ and $C_{D_{CZ}(k+1)} = [R_{D_{CA}(k)}, 0, R_{D_{CA}(k)}, 0]$. Now we only need to prove that for any vertex $p, l \in V(\omega(C_{D_{CZ}(k+1)}, u))$, $DB(p, l) \neq \varnothing$. Let $\omega(C_{D_{CZ}(k+1)}, u) = u[R_{D_{CA}(k)}]v[0]x[R_{D_{CA}(k)}]y[0]z$, by the induction hypothesis, we know that for any vertex $p, l \in V(u[R_{D_{CA}(k)}]v)$, $DB(p, l) \neq \varnothing$ and for any vertex $p_1, l_1 \in V(x[R_{D_{CA}(k)}]y)$, $DB(p_1, l_1) \neq \varnothing$. Because that u and x at least differs form the bit 0, so that we know that for any vertex $p, l \in V(\omega(C_{D_{CZ}(k+1)}, u))$, $DB(p, l) \neq \varnothing$ except u and z. By this result, Property 1 and Lemma 2, the theorem holds for $n = k + 1$. $\square$

Furthermore, by Property 1, Property 2 and Property 3, we have a conjecture about some permutation which is not a cycle permutation.

Conjecture 1. *Let $D(s) = (d_0, \ldots, d_{s-2}, d_{s-1})$ where $1 \leq s \leq n$. If $|d_{s-2} - d_{s-1}| = 1$ where $d_{s-2} \neq 0$ and $d_{s-1} \neq 0$, $C_{D(s)}$ is not a cycle sequence.*

5 Conclusion

The Möbius cubes MQ_n is a variant of the hypercube structure. In this paper, we propose a method of embedding Hamiltonian cycle into the Möbius cubes and further we have a conjecture that one kind of permutation which can't generate a cycle in the Möbius cubes. We are still looking for more useful permutations and proving our conjecture.

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