Connectivity of Locally Exchanged Twisted Cubes*

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Abstract

Connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal leaves the remaining graph disconnected or trivial. This parameter is important for interconnection networks and can be used to measure reliability in such networks. In this paper, a new interconnection network called locally exchanged twisted cube (LETQ for short), denoted $LeTQ(s,t)$, is proposed. We obtain some basic properties of LETQ including isomorphism, decomposition, hamiltonicity and connectivity. In particular, we determine $\kappa(LeTQ(s,t)) = \min\{s+1, t+1\}$.

Keyword: Interconnection networks; Exchanged hypercubes; Locally twisted cubes; Connectivity; Locally exchanged twisted cubes.

1 Introduction

Interconnection networks play an important role in parallel computing systems, and such a network can be modeled by a graph $G = (V,E)$, where V (or $V(G)$) represents the set of processors and E (or $E(G)$) is the set of communication links between processors. It is well-known that hypercubes are the most popular and efficient interconnection networks due to their rich topological properties, such as regularity, symmetry, small diameter, strong connectivity, recursive structure, flexible partition, and relatively low link complexity [21]. To overcome some shortcomings of a normal hypercube, variations of hypercube architecture have been proposed for achieving the improvement on their efficiency. For more previous results on variations of hypercubes, the reader can refer to [2, 6].

As a variant of hypercube, the family of locally twisted cubes was first introduced by Yang et al. [26]. It was shown that the diameter of an n-dimensional locally twisted cube, denoted as LTQ_n, is only about half of that of the corresponding hypercube. Another advantage is that the rule of adjacency for vertices in LTQ_n is simpler than other variations. In particular, any two adjacent vertices in LTQ_n differ only in at most two successive bits. More attractive properties and application support merits of LTQ_n can be found in the literature, e.g., studies on diagnosability [25], mesh embedding [3], hamiltonicity [5, 16, 17, 24, 27], and independent spanning trees [4, 11, 12].

Recently, exchanged hypercube $EH(s,t)$, proposed by Loh et al. [14] is a new interconnection network obtained by systematically removing links from a hypercube. The structure of exchanged hypercubes not only kept numerous desirable properties of the normal hypercubes, but also reduced the interconnection complexity. In particular, researches of exchanged hypercubes have been devoted on the topics including domination [7,8], connectivity [10,15,19], cycle embedding [18], edge congestion [23], wide and fault diameter [22], and others [1, 13]. In addition, a variant of exchanged hypercube called exchanged crossed cubes has been studied recently in [9, 20].

In this paper, we combine the notion of locally twisted cubes and exchanged hypercubes to introduce a new type of network topology called locally exchanged twisted cube (LETQ for short), which retains most well features of the two originals. Then, we obtain some basic properties of LETQ including isomorphism, decomposition, hamiltonicity and connectivity. Especially, connectivity is an important parameter for interconnection networks and it can be used to measure reliability in such networks.

The rest of this paper is organized as follows. Section 2 gives the definition of LETQ. Section 3

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provides some basic properties of LETQ. Section 4 proves the main results for finding the connectivity of LETQ. Finally, concluding remarks are given in Section 5.

2 Locally exchanged twisted cubes

For convenience, vertices in a hypercube or its variants are usually encoded by using binary string representation. Also, we use the notation G^x to denote the labeled graph obtained from a graph G by prefixing the binary string of every vertex with x. Let $\oplus$ denote the modulo 2 addition. The *n-dimensional locally twisted cube LTQ_n* is defined as follows:

(1) LTQ_2 is a graph consisting of four vertices with labels 00, 01, 10, 11 together with four edges (00,01), (00,10), (01,11), and (10,11).

(2) For $n \geqslant 3$, LTQ_n is constructed from two copies of LTQ_{n-1} (i.e., LTQ^0_{n-1} and LTQ^1_{n-1}) by the following rule: each vertex $x = 0x_{n-2}x_{n-3}\cdots x_0$ in LTQ^0_{n-1} is connected with the vertex $1(x_{n-2}\oplus x_0)x_{n-3}\cdots x_0$ in LTQ^1_{n-1} by an edge.

Inspired by the idea of exchanged hypercube, we pose the following definition.

Definition 1. A *locally exchanged twisted cube* is an undirected graph $LeTQ(s,t) = G(V,E)$, where $s,t \geqslant 1$, $V = \{x = x_{t+s}\cdots x_{t+1}x_t\cdots x_1x_0\colon x_i \in \{0,1\}$ for $0 \leqslant i \leqslant t+s\}$ is the vertex set, and E is the edge set composed of the following three types of disjoint sets E_1, E_2 and E_3:

$$E_1 = \{(x,y)\in V\times V\colon\ x\oplus y = 2^0\},$$

$$E_2 = \{(x,y)\in V\times V\colon\ x_0=y_0=1, x_1=y_1=0 \text{ and } x\oplus y=2^k \text{ for } k\in[3,t]\} \cup \{(x,y)\in V\times V\colon\ x_0=y_0=x_1=y_1=1 \text{ and } x\oplus y=2^k+2^{k-1} \text{ for } k\in[3,t]\} \cup \{(x,y)\in V\times V\colon\ x_0=y_0=1 \text{ and } x\oplus y\in\{2^1,2^2\}\},$$

and

$$E_3 = \{(x,y)\in V\times V\colon\ x_0=y_0=x_{t+1}=y_{t+1}=0 \text{ and } x\oplus y=2^k \text{ for } k\in[t+3,t+s]\} \cup \{(x,y)\in V\times V\colon\ x_0=y_0=0, x_{t+1}=y_{t+1}=1 \text{ and } x\oplus y=2^k+2^{k-1} \text{ for } k\in[t+3,t+s]\} \cup \{(x,y)\in V\times V\colon\ x_0=y_0=0 \text{ and } x\oplus y\in\{2^{t+1},2^{t+2}\}\}.$$

For convenience, we write $E_i(G)(=E_i)$ for $i \in \{1,2,3\}$ if we need to indicate the graph G. From the above definition, the binary string of a vertex in $LeTQ(s,t)$ is partitioned into three parts: s-part (i.e., the leftmost part with s bits), t-part (i.e., the middle part with t bits) and the rightmost bit. Accordingly, $LeTQ(s,t)$ contains 2^{s+t+1} vertices. Also, we have $|E_1| = 2^{s+t}$, $|E_2| = t2^{s+t-1}(= 2^s \times t2^{t-1})$ and $|E_3| = s2^{s+t-1}(= 2^t \times s2^{s-1})$. Thus, the total number of edges in $LeTQ(s,t)$ approaches one half of that in LTQ_{s+t+1} as s and t are larger. To be more precise, $|E(LeTQ(s,t))| = (s+t+2)2^{s+t-1} = (\frac{1}{2}+\frac{1}{2(s+t+1)})|E(LTQ_{s+t+1})|$. Figure 1 depicts the locally exchanged twisted cube $LeTQ(1,3)$, where links with label "0" correspond to the edge set E_1, links with label "i" for $i \in \{1,2,3\}$ correspond to the edge set E_2, and links with label "4" correspond to the edge set E_3.

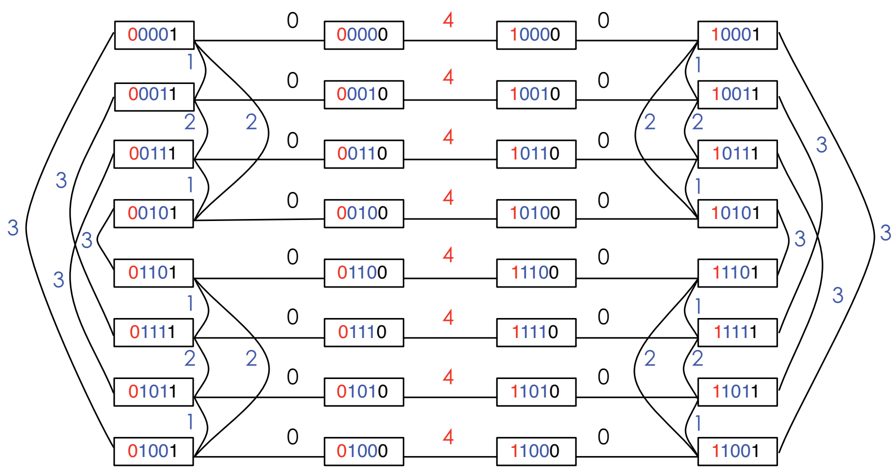

Figure 1: Locally exchanged twisted cube $LeTQ(1,3)$.

The *degree* of a vertex x in a graph G, denoted by $\deg_G(x)$, is defined to be the number of edges connected to x in G, where we omit the subscript G if the graph is clear from the context. Also, we write $\delta(G) = \min_{x \in G} \deg(x)$ to stand for the minimum degree of vertices in G. For $LeTQ(s,t)$, it is easy to verify that the degree of a vertex x with the rightmost bit 0 (respectively, the rightmost bit 1) is $s+1$ (respectively, $t+1$). Thus, $\delta(LeTQ(s,t)) = \min\{s+1, t+1\}$.

3 Basic Properties

Two graphs G and H are *isomorphic*, denoted by $G \cong H$, if there is a bijection ϕ: $V(G) \to V(H)$ such that $(x,y) \in E(G)$ if and only if $(\phi(x), \phi(y)) \in E(H)$. From the definition of LETQ, we note that $LeTQ(s,t) \cong EH(s,t)$ if and only if $s,t \leqslant 2$.

Lemma 1. *$LeTQ(s,t) \cong LeTQ(t,s)$.*

Proof. Let ϕ: $V(LeTQ(s,t)) \to V(LeTQ(t,s))$ be the bijection defined as follows:

$$x(= x_{t+s} \cdots x_{t+1} x_t \cdots x_1 x_0) \overset{\phi}{\longmapsto} x'(= x'_{s+t} \cdots x'_{s+1} x'_s \cdots x'_1 x_0)$$

where

$$\begin{cases} x'_i = x_{t+i} & \text{for } 1 \leqslant i \leqslant s; \\ x'_{s+i} = x_i & \text{for } 1 \leqslant i \leqslant t. \end{cases}$$

By definition, we have $(x,y) \in E_1(LeTQ(s,t)) \Leftrightarrow x \oplus y = 2^0 = \phi(x) \oplus \phi(y) \Leftrightarrow (\phi(x), \phi(y)) \in E_1(LeTQ(t,s))$. Also, it is not hard to verify from the bijection that $(x,y) \in E_2(LeTQ(s,t)) \Leftrightarrow (\phi(x), \phi(y)) \in E_3(LeTQ(t,s))$ and $(x,y) \in E_3(LeTQ(s,t)) \Leftrightarrow (\phi(x), \phi(y)) \in E_2(LeTQ(t,s))$. Thus, the two graphs are isomorphic. $\square$

In [14], it has been shown that an exchanged hypercube $EH(s,t)$ can be decomposed into two copies of $EH(s-1,t)$ or $EH(s,t-1)$. According to the definition of LETQ, the following lemma can be proven by using the same way of the decomposition of $EH(s,t)$.

Lemma 2. *$LeTQ(s,t)$ can be decomposed into two disjoint copies of $LeTQ(s-1,t)$ when $s \geqslant 2$ or $LeTQ(s,t-1)$ when $t \geqslant 2$ such that the two copies of subgraph are connected by 2^{s+t-1} edges in $LeTQ(s,t)$.*

Proof. We first consider $t \geqslant 2$ and choose an integer $k \in [1,t]$. Let

$$\tilde{E} = \begin{cases} \{(x,y) \in V \times V \colon x \oplus y = 2^k \text{ and } x_1 = y_1 = 0\} & \text{if } k \in [3,t]; \\ \{(x,y) \in V \times V \colon x \oplus y = 2^k + 2^{k-1} \text{ and } x_1 = y_1 = 1\} & \text{if } k \in [3,t]; \\ \{(x,y) \in V \times V \colon x \oplus y = 2^k\} & \text{if } k \in [1,2]. \end{cases}$$

Clearly, $\tilde{E}$ is contained in E_2 and the removal of $\tilde{E}$ from $LeTQ(s,t)$ results in two disjoint copies of $LeTQ(s,t-1)$. On the other hand, if $s \geqslant 2$, the lemma can be proved by considering $k \in [t+1, t+s]$ and let

$$\tilde{E} = \begin{cases} \{(x,y) \in V \times V \colon x \oplus y = 2^k \text{ and } x_{t+1} = y_{t+1} = 0\} & \text{if } k \in [t+3, t+s]; \\ \{(x,y) \in V \times V \colon x \oplus y = 2^k + 2^{k-1} \text{ and } x_{t+1} = y_{t+1} = 1\} & \text{if } k \in [t+3, t+s]; \\ \{(x,y) \in V \times V \colon x \oplus y = 2^k\} & \text{if } k \in [t+1, t+2]. \end{cases}$$

Similarly, $\tilde{E}$ is contained in E_3 and there are two disjoint copies of $LeTQ(s-1,t)$ when $\tilde{E}$ is removed from $LeTQ(s,t)$. Since $|E(LeTQ(s,t))| = (s+t+2)2^{s+t-1}$, we have

$$|\tilde{E}| = (s+t+2)2^{s+t-1} - 2((s+t+1)2^{s+t-2}) = 2^{s+t-1}.$$

$\square$

Hamiltonicity is an important property for data transmission in interconnection networks. In [14], exchanged hypercube $EH(s,t)$ has been proved to be hamiltonian, i.e., it contains a cycle passing through every vertex exactly once. For more properties related to hamiltonicity of $EH(s,t)$, the reader can also refer to [18]. In what follow, we use 0^k to stand for a sequence of k 0s (i.e., $\underbrace{0 \cdots 0}_{k}$).

Lemma 3. *$LeTQ(s,t)$ is hamiltonian for $s,t \geqslant 1$.*

Proof. By Lemma 1, as $LeTQ(s,t) \cong LeTQ(t,s)$, we may assume $s \leqslant t$. The proof is by induction on t. Since $LeTQ(s,t) \cong EH(s,t)$ for $t \leqslant 2$, the existence of a hamiltonian cycle has been proved in [14]. We now consider $t \geqslant 3$ and suppose that $LeTQ(s,t-1)$ is hamiltonian. According to Lemma 2, we decompose $LeTQ(s,t)$ into two subgraphs G_0 and G_1, where each subgraph G_k for $k \in \{0,1\}$ is isomorphic to $LeTQ(s,t-1)$ with the vertex set V_k shown below:

$$V_k = \{x_{t+s} \cdots x_{t+1} k x_{t-1} \cdots x_1 x_0 \colon x_i \in \{0,1\} \text{ for } i \in [0,t-1] \cup [t+1, t+s]\}.$$

Based on induction hypothesis, there exists a hamiltonian cycle R_0 in G_0 that contains the edge (x,y) for $x = 0^s000^{t-2}1$ and $y = 0^s010^{t-2}1$. Similarly, there exists a hamiltonian cycle R_1 in G_1

that contains the edge (z, w) for $z = 0^s 100^{t-2}1$ and $w = 0^s 110^{t-2}1$. Since $(x, z), (y, w) \in E_2$, we can find a hamiltonian cycle that is obtained from the concatenation of R_0 and R_1 by replacing edges (x, y) and (z, w) with edges (x, z) and (y, w). $\square$

4 Proof of Connectivity

In this section, we first introduce some notations which will be used in the following proof. Let $G = (V, E)$ be a graph. For $S \subset V$ and $v \in V$, the set vertices adjacent to v and lying in S is denoted by $N_S(v)$. We use $G - S$ to denote the graph obtained from G by removing S. In particular, S is called a *vertex-cut* of G if $G - S$ is disconnected, and S is called a *super vertex-cut* of G if $G - S$ is disconnected and without isolated vertex. The *connectivity* (respectively, *edge-connectivity*) of a graph G, denoted by $\kappa(G)$ (respectively, $\lambda(G)$), is the minimum number of vertices (respectively, edges) whose removal leaves the remaining graph disconnected or trivial. It is well known that $\kappa(G) \leqslant \lambda(G) \leqslant \delta(G)$. Moreover, the *super connectivity* $\kappa'(G)$ (respectively, *super edge-connectivity* $\lambda'(G)$) is the minimum number of vertices (respectively, edges) whose removal results in a disconnected graph without isolated vertex.

Theorem 4. $\kappa(LeTQ(s,t)) = \lambda(LeTQ(s,t)) = \min\{s+1, t+1\}$ *for* $s, t \geqslant 1$. *Moreover, every minimum vertex-cut is the set of neighbors of some vertex in* $LeTQ(s,t)$.

Proof. By Lemma 1, without loss of generality we assume $s \leqslant t$, and thus $\kappa(LeTQ(s,t)) \leqslant \delta(LeTQ(s,t)) = \min\{s+1, t+1\} = s+1$. In the following, we will show that $\kappa(LeTQ(s,t)) \geqslant s+1$. By Lemma 3, $\kappa(LeTQ(s,t)) \geqslant 2$. For $s = 1$ (see e.g. Figure 1), since $\delta(LeTQ(1,t)) = s+1 = 2$, it implies $\kappa(LeTQ(1,t)) = 2$. We now suppose $s \geqslant 2$. The proof is by induction on $s+t$. Since $LeTQ(2,2)$ is isomorphic to $EH(2,2)$, $\kappa(EH(2,2)) = 3$ has been proved in [15]. We consider $s+t \geqslant 5$ and suppose that $\kappa(LeTQ(s-1,t)) = \min\{s, t+1\}$ and $\kappa(LeTQ(s,t-1)) = \min\{s+1, t\}$. We partition $LeTQ(s,t)$ into two subgraphs L and R as follows:

$$V(L) = \{0x_{t+s-1} \cdots x_{t+1}x_t \cdots x_1x_0 \colon x_i \in \{0,1\} \text{ for } 0 \leqslant i \leqslant t+s-1\}$$

and

$$V(R) = \{1x_{t+s-1} \cdots x_{t+1}x_t \cdots x_1x_0 \colon x_i \in \{0,1\} \text{ for } 0 \leqslant i \leqslant t+s-1\}.$$

By Lemma 2, both L and R are isomorphic to $LeTQ(s-1,t)$. Let $\tilde{E}$ be the set of edges connecting L and R. Clearly, $\tilde{E} \subset E_3$. Moreover, we subdivide $V(L)$ into A and B such that A (respectively, B) contains vertices with the rightmost bit 0 (respectively, the rightmost bit 1). Similarly, we subdivide $V(R)$ into C and D such that C (respectively, D) contains vertices with the rightmost bit 0 (respectively, the rightmost bit 1). Thus, the edges between A and B (respectively, C and D) lie in E_1 and form a perfect matching in L (respectively, in R).

Assume that S is a minimum vertex-cut of $LeTQ(s,t)$. Let $S_L = S \cap V(L)$ and $S_R = S \cap V(R)$. If both $L - S_L$ and $R - S_R$ are connected, then every edge $e \in \tilde{E}$ must have at least one endvertex in S. By Lemma 2, this yields $|S| \geqslant |\tilde{E}| = 2^{s+t-1} > s+t-1 \geqslant s+1$, a contradiction. Hence, we may assume that $L - S_L$ is disconnected. This implies, by induction hypothesis, $|S_L| \geqslant \kappa(L) = \kappa(LeTQ(s-1,t)) = \min\{s, t+1\} = s$. If $R - S_R$ is also disconnected, then $|S_R| \geqslant \kappa(R) = s$. It follows that $|S| = |S_L| + |S_R| \geqslant 2s > s+1$, a contradiction. Hence, $R - S_R$ is connected. Consider the following two subcases:

Case 1: $S_R = \emptyset$. Clearly, $S = S_L$ and R is connected. Let $M = S \cap A$ and $F = S \cap B$. Also, let M' be the subset of B such that every vertex of M' is adjacent to a vertex of M by an edge in E_1 (see Figure 2). Then $|M'| = |M|$. Let T be the subgraph obtained from $LeTQ(s,t)$ by removing $S \cup M'$. It is clear that $V(T) = V(R) \cup (A - M) \cup (B - F - M')$. Since each vertex of $B - F - M'$ is adjacent to a vertex in $A - M$ by an edge in E_1 and each vertex of $A - M$ is adjacent to a vertex in C (and thus in R) by an edge in $\tilde{E}$, the subgraph T is connected. Hence, removing S from $LeTQ(s,t)$ make some vertex $v \in M'$ disconnect to T. Then, we have $N_B(v) \subseteq F \cup M'$. Otherwise, there is a vertex $w \in B - F - M'$ such that v is connected to T through w. Thus, $|N_B(v)| = |N_F(v)| + |N_{M'}(v)| \leqslant |F| + |M'| - 1$. Since $v \in B$, we have $|N_M(v)| = 1$ and $|N_B(v)| = t$. This shows that $\kappa(LeTQ(s,t)) = |S| = |F| + |M| = |F| + |M'| \geqslant |N_B(v)| + 1 = t+1 \geqslant s+1$.

Next, we claim that $|M'| = 1$, i.e., there is only one vertex $v \in M'$ disconnecting to T after removing S from $LeTQ(s,t)$. Suppose not, and there exists another vertex $u (\neq v) \in M'$ such that u and v are adjacent. Because there is no cycle of length three in $LeTQ(s,t)$, u and v have no common neighbors. Also, from the above argument, we know that $N_B(u) \cup N_B(v) \subseteq F \cup M'$. Thus, $|S| = |F| + |M| = |F| + |M'| \geqslant |N_B(u)| + |N_B(v)| = 2t > t+1 \geqslant s+1$, a contradiction. On the other hand, we consider $|M'| \geqslant 2$ and every vertex $u (\neq v) \in M'$ is nonadjacent to v. Since $N_B(v) \subseteq F$, we have $|S| = |F| + |M| \geqslant |N_B(v)| + |M'| = t+2 > s+1$, a contradiction. According to the aforementioned claim, S is the set of neighbors of v in $LeTQ(s,t)$.

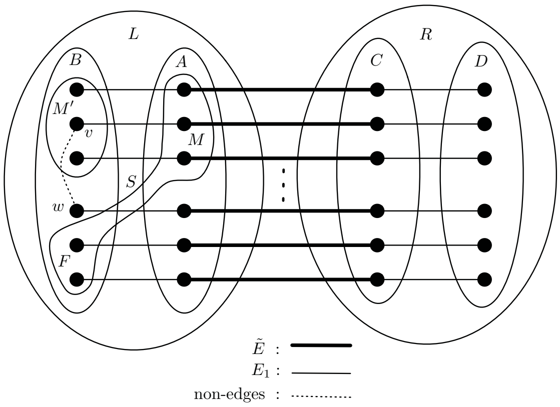

Figure 2: Illustration for the proof of Case 1 in Theorem 4.

Case 2: $S_R \neq \emptyset$. That is, $|S_R| \geqslant 1$. By induction hypothesis, $|S_L| \geqslant \kappa(L) = \kappa(LeTQ(s-1,t) = s$ and a minimum vertex-cut of L is the set of neighbors of some vertex in L. Thus, we have $\kappa(LeTQ(s,t)) = |S| = |S_L| + |S_R| \geqslant s+1$. Since $L - S_L$ is disconnected and $R - S_R$ is connected, by induction hypothesis, equality requires S_L to be the set of neighbors of some vertex $v \in A$ (see Figure 3). Hence, the unique method for removing a vertex to break all paths from v to R is to remove the neighbor of v in R.

Accordingly, we complete the proof. □

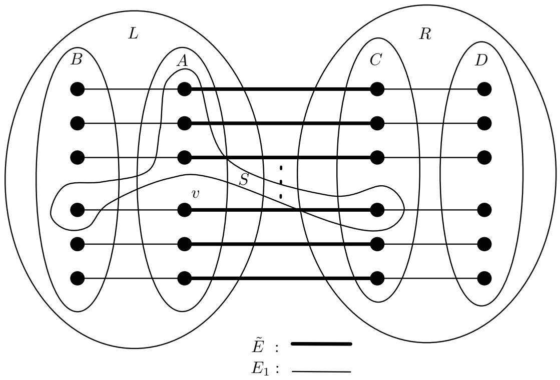

Figure 3: Illustration for the proof of Case 2 in Theorem 4.

5 Concluding remarks

In this paper, we introduce a new interconnection network called locally exchanged twisted cube. This network retains most of the well topological features of locally twisted cube and exchanged hypercube. Some basic properties including isomorphism, decomposition, hamiltonicity and connectivity of LETQ are provided. In particular, we obtain $\kappa(LeTQ(s,t)) = \min\{s+1, t+1\}$.

The *super connectivity* of a network is the minimum number of vertices whose removal leaves the remaining network disconnected and without isolated vertex. Again, this parameter is important for measuring reliability of a communication network. Thus, it would be an interesting study for finding the super connectivity of LETQ in the future research.

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