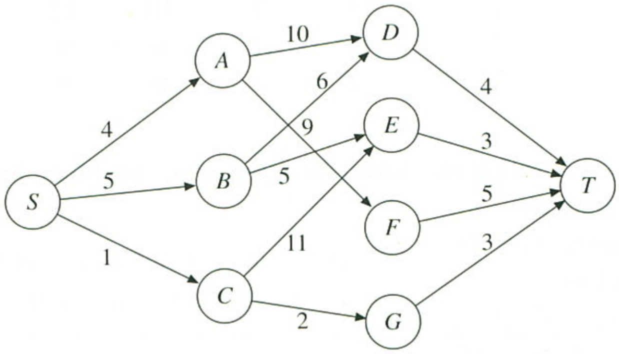
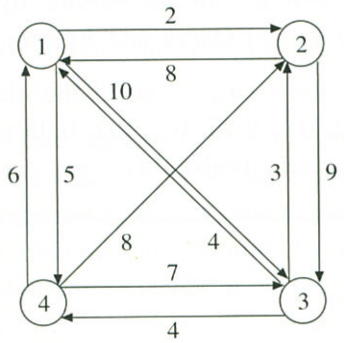
Exercises

7.1 Consider the following graph. Find the shortest route from S to T by the dynamic programming approach.



7.2 For the graph shown in Figure 7–1, solve the same problem by using the branch-and-bound approach. For this problem, which approach (dynamic programming versus branch-and-bound) is better? Why?

7.3 For the graph shown as follows, solve the traveling salesperson problem by the branch-and-bound approach. Compare it with the dynamic programming approach.



	1	2	3	4
1	∞	2	10	5
2	8	∞	9	∞
3	4	3	∞	4
4	6	8	7	∞

7.4 For the following table, find an optimal allocation of resources to maximize the total profit for those three projects and four resources.

project \ resource	1	2	3	4
1	3	7	10	12
2	1	2	6	9
3	2	4	8	9

7.5 Solve the following linear programming problem by dynamic programming.
Maximize $x_0 = 8x_1 + 7x_2$
subject to
$2x_1 + x_2 \le 8$
$5x_1 + 2x_2 \le 15$
where x_1 and x_2 are non-negative integers.

7.6 Find a longest common subsequence of

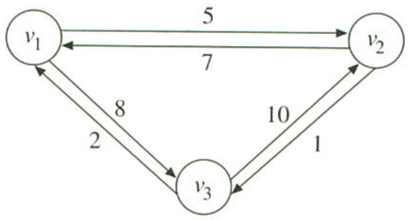
$S_1 = a\ a\ b\ c\ d\ a\ e\ f$

and $S_2 = b\ e\ a\ d\ f.$

7.7 In general, the partition problem is NP-complete. However, under some constraints, a special kind of the partition problem is a polynomial problem because it can be solved by dynamic programming. Read Section 4–2 of Garey and Johnson (1979).

7.8 Find an optimal binary tree for $a_1, a_2, \ldots, a_6$, if the identifiers, in order, have probabilities 0.2, 0.1, 0.15, 0.2, 0.3, 0.05 respectively and all other identifiers have zero probability.

7.9 Consider the following graph: Solve the all-pairs shortest paths problem of the graph. The all-pairs shortest paths problem is to determine the shortest path between every pair of vertices. Consult Section 5–3 of Horowitz and Sahni (1978), or Section 5–4 of Brassard and Bratley (1988).



7.10 Let f be a real function of x and $y = (y_1, y_2, \ldots, y_k)$. We say that f is decomposable into f_1 and f_2 if f is separable $(f(x, y) = f_1(x), f_2(y))$ and if, moreover, the function is monotone non-decreasing relative to its second argument. Prove that if f is decomposable with $f(x, y) = (f_1(x), f_2(y))$, then

$$\underset{(x,y)}{Opt}\{f(x, y)\} = Opt\{f_1(x, Opt\{f_2(y)\})\} \quad (Opt = \text{min or max})$$

(Consult Section 9–2 of [Minoux 1986].)

7.11 Floyd's algorithm, which can be easily found in many textbooks, is to find all-pairs shortest paths in a weighted graph. Give an example to explain the algorithm.

7.12 Write a dynamic programming algorithm to solve longest increasing subsequence problem.

7.13 Given two sequences S_1 and S_2 on a alphabet set Σ, and a scoring function $f: \Sigma \times \Sigma \rightarrow \Re$, the local alignment problem is to find a subsequence S_1' from S_1 and a subsequence S_2' from S_2 such that the score obtained by aligning S_1' and S_2' is the highest, among all possible subsequences of S_1 and S_2. Use the dynamic programming strategy to design an algorithm of $O(nm)$ time or better for this problem, where n and m denote the lengths of S_1 and S_2 respectively.