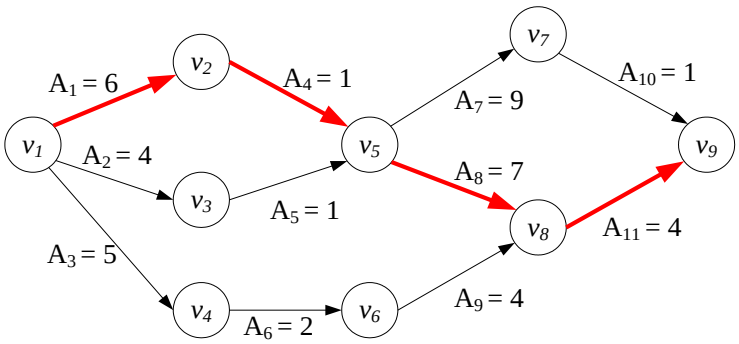
# Department of Computer Science and Engineering
# National Sun Yat-sen University
# Advanced Programming and Practice - Final Exam., June 9, 2022

1. Explain each of the following terms. (20%)
   (a) convex hull
   (b) longest common subsequence
   (c) 0/1 knapsack problem
   (d) Best-first search
   (e) hill climbing

2. (a) What is the *Eulerian circuit* in a graph? (5%)
   (b) What is sufficient and necessary condition for that there exists an *Eulerian circuit* in a graph? (5%)

3. The *critical path* problem is to find the longest path in an acyclic directed graph. For example, in the below figure, the answer is $v_1$, $v_2$, $v_5$, $v_8$, $v_9$, whose total length is 18. Please present an algorithm for solving it with the below figure as an example. (10%)




4. (a) Explain the *2D maxima* problem on the 2D plane. Please give an example to describe your answer. (5%)
   (b) Present a *divide-and-conquer* algorithm for solving the above problem. And analyze the time complexity of your algorithm. (10%)

5. Suppose there are seven symbols A, B, C, D, E, F, G in a file with occurrences 3, 4, 5, 6, 13, 15, 16, respectively. Please give the Huffman code tree constructed by Huffman algorithm. (15%)

6. Let P($n$,$m$) denote the problem of $m$-permutations selected from {1,2,…,$n$} without repetition. These $m$-permutations are represented with the *lexicographic order*. For example, in P(5,4), we have…, 4312, 4315, 4321… The *rank* of an $m$-permutation $c$ in P($n$,$m$), denoted as $r(c)$, is the number of permutations before $c$ in the

lexicographic order. Note that the first permutation 1234 in P(5,4) is ranked as 0, that is $r(1234)=0$. Answer the following questions for P(8,5).
(a) What is the next one of 78654? (5%)
(b) What is $r(24635)$? Explain the way of your calculation. (5%)
(c) Please give the permutation $c$ such that $r(c)=500$. Explain the way of your calculation. (5%)

7. In the *coin change* problem (CCP), you are given four types of coins, 1-cent (type 1), 2-cent (type 2), 5-cent (type 3), and 10-cent (type 4), with unlimited amounts. Suppose that you have money $n$ cents. How many are there different ways to exchange $n$ cents with these coins? CCP can be solved by the dynamic programming method. Let $m(i,j)$ denote the count for changing $j$ cents with coin type $i' \le i \le 4$. For $j$ between 1 and 20, the table of $m(i,j)$ is shown as follows.
(a) Please fill in all the blanks in the table (5%).
(b) Please give the dynamic programming formula for solving $m(i,j)$. Here, you can assume that c(1)=1, c(2)=2, c(3)=5, and c(4)=10 are known. (10%)

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m(1,j)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $m(2,j)$ | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | | | | | | | |
| $m(3,j)$ | 1 | 2 | 2 | 3 | 4 | | | | | | | | | | | | | | | |
| $m(4,j)$ | 1 | 2 | 2 | 3 | 4 | | | | | | | | | | | | | | | |

Answers:
6. (a) 81234 (b) 1145 (c) 16324

7. (a)

| $j$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m(1,j)$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $m(2,j)$ | 1 | 2 | 2 | 3 | 3 | 4 | 4 | 5 | 5 | 6 | 6 | 7 | 7 | **8** | **8** | **9** | **9** | **10** | **10** | **11** |
| $m(3,j)$ | 1 | 2 | 2 | 3 | 4 | **5** | **6** | **7** | **8** | **10** | **11** | **13** | **14** | **16** | **18** | **20** | **22** | **24** | **26** | **29** |
| $m(4,j)$ | 1 | 2 | 2 | 3 | 4 | **5** | **6** | **7** | **8** | **11** | **12** | **15** | **16** | **19** | **22** | **25** | **28** | **31** | **34** | **40** |

(b)
m(1,j)=1, m(i,0)=1,
m(i,j)=m(i-1,j)+m(i, j-c(i)), for i ≥ 2 and j ≥ 1