Department of Computer Science and Engineering
National Sun Yat-sen University
Advanced Programming and Practice - Final Exam., June 1, 2023

1. Explain each of the following terms. (20%)
   (a) transitive closure in a graph
   (b) hill climbing
   (c) topological order
   (d) traveling salesperson problem
   (e) branch and bound

2. (a) What will be printed by the following C program? (5%)
   ```
   printf("%d", (-24)&24);    // &: bitwise AND
   ```
   (b) What is the purpose of (-x) & x for a positive integer x? (5%)

3. (a) Present an algorithm for finding the minimum spanning tree of a given graph. (10%)
   (b) Analyze the time complexity of your algorithm. (5%)

4. (a) Explain the *2D maxima* problem on the 2D plane. Please give an example to describe your answer. (5%)
   (b) Present a *divide-and-conquer* algorithm for solving the above problem. And analyze the time complexity of your algorithm. (10%)

5. In the *matrix-chain multiplication* problem, we are given n matrices A_1, A_2, …, A_n with size $p_0 \times p_1$, $p_1 \times p_2$, $p_2 \times p_3$,…, $p_{n-1} \times p_n$. It is well-known that the computation of $A_i \times A_{i+1}$ needs $p_{i-1} \times p_i \times p_{i+1}$ scalar multiplications. The problem is to determine the multiplication order such that the number of scalar multiplications is minimized. It can be solved by the dynamic programming (DP) approach. Let $m(i,j)$ denote minimum number of scalar multiplications for computing $A_i \times A_{i+1} \times \ldots \times A_j$. Please give the DP formula for solving this problem. (15%)

6. Let C(n,m) denote m-combinations selected from {1,2,…,n} without repetition. Each m-combination is represented with *lexicographic order*. For example, the two combinations 251 and 512 are the same, and they are represented with 125. These m-combinations in C(n,m) are represented with the *lexicographic order*. For example, all 3-combinations selected from {1,2,3,4} in lexicographical order are 123, 124, 134, 234. The *rank* of an m-combination c in C(n,m), denoted as $r(c)$, is the number of combinations before c in the lexicographic order. Note that the first permutation 1234 in C(5,4) is ranked as 0, that is r(1234)=0. Answer the following questions for C(9,5).

(a) What is the next one of 12489? (5%)
(b) What is $r(24578)$? Explain the way of your calculation. (5%)
(c) Which combination has the rank 80? Explain how do you calculate? (5%)

7. Given a sequence of integers $a_1, a_2,\ldots,a_n$ (positive, zero, or negative), the *maximum segment* problem aims to find a segment $a_i, a_{i+1},\ldots,a_j$, $1 \le i \le j \le n$, such that $a_i+a_{i+1}+\ldots+a_j$ has the maximum value. In the given sequence, at least one integer is positive. Please design an algorithm for solving the problem in linear time. Analyze the time complexity. Note that you will get no point if the time complexity of your algorithm is not linear. (10%)

Answers:
2. (a) 8
(b) indicate the rightmost bit of value 1 for *x*

6. Combination
(a) 12567
(b) C(9,5)=(9!)/((5!)*(4!))=126
The maximum rank is 126-1=125, since the first rank is 0.
Rank of 24578:
125-[C(7,5)+C(5,4)+C(4,3)+C(2,2)+C(1,1)]
=125-[21+5+4+1+1]
=126-32
=93
(b) Rank 80
The maximum rank is 125.
# of combinations greater than rank 80: 125-80=45

C(8,5)=56 > 45 ≥ C(7,5)=21, select C(7,5), thus first digit is 2.
45-21=34
34 ≥ C(6,4)=30, thus second digit is 3.
34-30=4
4 ≥ C(4,3)=4, thus third digit is 5.
4-4=0
0 ≥ C(1,2)=0, thus fourth digit is 6.
0-0=0
0 ≥ C(0,1)=0, thus fifth digit is 7.
Answer: 23567.