

Department of Computer Science and Engineering
National Sun Yat-sen University
Advanced Programming and Practice - Final Exam., June 6, 2024

1. Explain each of the following terms. (20%)
 - (a) Euler circuit in a graph
 - (b) best first search
 - (c) sum of subset problem
 - (d) coloring problem in a graph
 - (e) Huffman codes
2. (a) What is the *knapsack* problem? (5%)
(b) Design a *greedy* algorithm for solving the above problem. (10%)
3. (a) Present an algorithm for finding all 2-D *maxima* in a set of 2-D points. (10%)
(b) Analyze the time complexity of your algorithm. (5%)
4. (a) Explain the *longest common subsequence* problem. Please give an example to describe your answer. (5%)
(b) Present a *dynamic programming* method for solving the above problem. And analyze the time complexity of your algorithm. (10%)
5. (a) Let $C(n,m)$ denote m -combinations selected from $\{1,2,\dots,n\}$ without repetition. Each m -combination is represented with *lexicographic order*. For example, the two combinations 251 and 512 are the same, and they are represented with 125. These m -combinations in $C(n,m)$ are represented with the *lexicographic order*. For example, all 3-combinations selected from $\{1,2,3,4\}$ in lexicographical order are 123, 124, 134, 234. What is the next one of 13589 in $C(9,5)$? (5%)
(b) Let $P(n,m)$ denote m -permutations selected from $\{1,2,\dots,n\}$ without repetition. The m -permutations in $P(n,m)$ are represented with the *lexicographic order*. What is the next one of 81397 in $P(9,5)$? (5%)
6. The logic-OR and logic-AND operations can be implemented by the linear constraints. For example, the implementation of $w = x \text{ OR } y$ is
$$\begin{aligned}w &\geq x \\w &\geq y \\w &\leq x + y \quad (+: \text{ integer addition}) \\w, x, y &\in \{0,1\}.\end{aligned}$$
With the similar concept, please present the implementation of $w = x \text{ AND } y$. (10%)
7. In the *range minimum query* (RMQ) problem, we are given a list of integers $T = \langle t_1, t_2, \dots, t_n \rangle$, where the elements in T are not sorted. The goal of the problem is to build

a data structure during the preprocessing stage so that the minimum value of a specified range can be determined efficiently during the query stage. For example, consider $T = \langle 5, 3, 10, 7, 8, 5, 9, 4 \rangle$ and $n=8$. The answer of RMQ for the range $[2, 5]$ is 3, which is the minimum of 3, 10, 7, 8 (i.e. t_2, t_3, t_4, t_5). As more examples, the answers of RMQ for $[1, 5]$, $[3, 5]$ and $[5, 7]$ are 3, 7 and 5, respectively. Please design an algorithm for solving this problem such that the preprocessing stage requires $O(n)$ time and each query requires $O(\log n)$ time. You have to analyze the time complexity of your algorithm. (15%)

Answers:

5. (a) 13678

(b) 81423

6. logic-AND

$$w \leq x$$

$$w \leq y$$

$$w \geq x + y - 1 \text{ (+: integer addition)}$$

$$w, x, y \in \{0,1\}.$$

<https://cs.stackexchange.com/questions/12102/express-boolean-logic-operations-in-zero-one-integer-linear-programming-ilp>

7. 採用 segment tree (線段樹)。細節請查詢 網路說明。