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Edge-bipancyclicity of star graphs under edge-fault tolerant ☆

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Abstract

The star graph S_n is one of the most famous interconnection networks. It has been shown by Li [T.-K. Li, Cycle embedding in star graphs with edge faults, Appl. Math. Comput. 167 (2005) 891–900] that S_n contains a cycle of length from 6 to $n!$ when the number of fault edges in the graph does not exceed $n-3$. In this paper, we improve this result by showing that for any edge subset F of S_n with $|F| \leqslant n-3$ every edge of $S_n - F$ lies on a cycle of every even length from 6 to $n!$ provided $n \geqslant 3$.

Keywords: Cycles; Star graph; Fault tolerant

1. Introduction

In interconnection networks, the problem of simulating one network by another is modelled as a graph embedding problem. There are several reasons why such an embedding is important [11]. For example, there are a number of efficient algorithms for solving some application problems and best communication patterns for their executions. For these algorithms, the existence of certain topological structures guarantee the desired performance. Thus, for such applications, it is desired to provide logically a specific topological structure throughout the execution of the algorithm in the network design.

Among all embedding problems, cycle embedding problem is one of the most popular problems, that is, finding a cycle of given length in a graph. A graph G is called *pancyclic* [3] if there exists a cycle of every length from 3 to $|V(G)|$. A graph is *bipartite graph* if its vertex-set can be partitioned into two disjoint subsets such that each edge is incident to two vertices from different subsets. A bipartite graph G is called *bipancyclic* if there exists a cycle of every even length from 4 to $|V(G)|$. The pancyclicity is an important metric in embedding cycles of any length into the topology of network. The concept of pancyclicity was extended to vertex-pancyclicity by Hobbs [6] and edge-pancyclicity by Alspach and Hare [2]. A graph G is called *vertex-pancyclic* if for

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any vertex u, there exists a cycle of every length from 3 to $|V(G)|$ containing u; and *edge-pancyclic* if for any edge e, there exists a cycle of every length from 3 to $|V(G)|$ containing e. Obviously, every edge-pancyclic graph is vertex-pancyclic. A bipartite graph G is *vertex-bipancyclic* if for any vertex u, there exists a cycle of every even length from 4 to $|V(G)|$ containing u. Similarly, a bipartite graph G is called *edge-bipancyclic* if for any edge e, there exists a cycle of every even length from 4 to $|V(G)|$ containing e. A graph G is said to be *Hamiltonian connected* if there exists a Hamiltonian path between any two vertices of G. It is easy to see that any bipartite graph with at least three vertices is not Hamiltonian connected. For this reason, Simmons [10] introduced the concept of Hamiltonian laceable for Hamiltonian bipartite graphs. A Hamiltonian bipartite graph is *Hamiltonian laceable* if there is a Hamiltonian path between any two vertices in different bipartite sets. Obviously, a Hamilton cycle can be embedded in the Hamiltonian connected graphs. Then the Hamiltonian connectivity is also important metric in embedding Hamitonian cycles into the topology of network. Since some components in a network would sometimes fail, it's more practical to study graphs with faults.

Star graphs, proposed by Akers and Krishnamurthy [1], is a famous interconnection networks. In this paper, we explore the embedding problems on star graphs. They proved that the star graphs are Cayley graphs, thus they are vertex symmetric. Furthermore, the star graphs have many other nice properties such as recursiveness, edge-symmetry [1]. Since the star graphs are bipartite graphs, odd cycles cannot be embedded into it. Jwo et al. [7] showed that any cycle of even length from 6 to $n!$ can be embedded into S_n. Hsieh et al. [5] and Li et al. [9], proved that the n-dimensional star graph S_n is $(n-3)$-edge-fault tolerant Hamiltonian laceable for $n \geqslant 4$. Recently, Li [8] considered the edge-fault tolerance of star graphs and showed that cycles of even length from 6 to $n!$ can be embedded into the n-dimensional star graphs when the number of the fault edges are less than $n-3$. In this paper, we improve this result by showing that for any edge subset F of S_n with $|F| \leqslant n-3$ and any edge $e \in S_n - F$, there exists a cycle of even length from 6 to $n!$ in $S_n - F$ containing e provided $n \geqslant 3$.

The rest of this paper is organized as follows. In Section 2, we give the definition and basic properties of the n-dimensional star graph S_n. In Section 3, we discuss the edge-fault-tolerant edge-bipancyclicity of the star graphs.

2. Star graphs

In this section, we give the definition and some properties of the star graphs. We follow [4] for graph-theoretical terminologies and notations not defined here.

The n-dimensional star graph, denoted by S_n, is a bipartite graph. The vertex-set is $V(S_n) = \{v|v \text{ is a permutation of } 1,2,\ldots,n\}$ and the edge-set is $E(S_n) = \{(u,v)|u = u_1u_2\cdots u_i\cdots u_n,\ v = u_iu_2\cdots u_{i-1}u_1\ u_{i+1}\cdots u_n\}$. Fig. 1 shows the four-dimensional star graph where the black vertices and the white vertices make the desired partition of vertex-set. There are some nice properties about the star graphs.

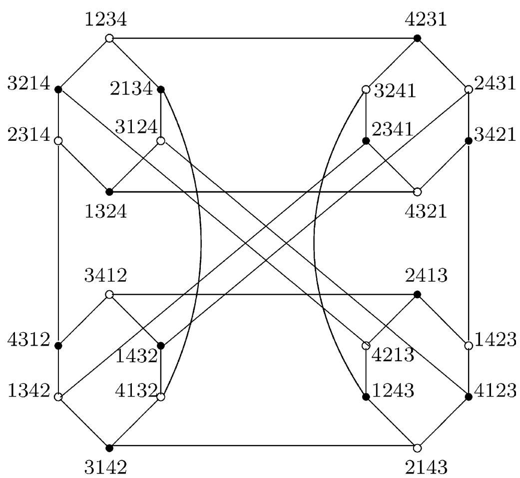

Fig. 1. Four-dimensional star graph S_4.

Lemma 1 (Li [8]). *There are n vertex-disjoint S_{n-1}'s in S_n for $n \geqslant 2$.*

Indeed, let $H^{i:j} = (V^{i:j}, E^{i:j})$, where $V^{i:j} = \{u \in V(S_n) | u = u_1u_2 \cdots u_i \cdots u_n, u_i = j\}$ and $E^{i:j} = \{(u,v) \in E(S_n) | u, v \in V^{i:j}\}$ for $1 \leqslant j \leqslant n$. Then $\{V^{i:1}, V^{i:2}, \ldots, V^{i:n}\}$ is a partition of $V(S_n)$ and $H^{i:j}$ is isomorphic to S_{n-1}. For convenience, we will use $S^{i:j}_{n-1}$ to denote the subgraph $H^{i:j}$ in the above partition. Specifically, we use S^{j}_{n-1} as the abbreviation of $S^{n:j}_{n-1}$ and call it the jth $(n-1)$-dimensional subgraph of S_n. For a vertex x, we use x_i to denote the ith digits of vertex x. For a set of distinct edges $e_1 = (x^1, y^1)$, $e_2 = (x^2, y^2), \ldots, e_m = (x^m, y^m)$ in S^i_{n-1} for $n \geqslant 3$ with $x^1_1 = x^2_1 = \cdots = x^m_1 = j$ and $y^1_1 = y^2_1 = \cdots = y^m_1 = k$, we call the edge set $\{e_1, e_2, \ldots, e_m\}$ *a set of* (i,j,k)*-edges*. Obviously, a set of (i,j,k)-edges is also a set of (i,k,j)-edges since the edges we discuss in this paper have no direction.

The following theorem was independently proved by Hsieh et al. [5] and Li et al. [9].

Theorem 2 (Hsieh et al. [5], Li et al. [9]). *The n-dimensional star graph S_n is $(n-3)$-edge-fault Hamiltonian laceable for $n \geqslant 4$.*

Lemma 3. *For any edge e and e' of S_4, there exists a cycle of even length from 6 to 24 in $S_4 - e'$ containing e.*

Proof. Since S_n is edge-symmetric, without loss of generality, we assume that $e = (1234, 3214)$. By Theorem 2, there exists a Hamiltonian path P connecting the vertices 1234 and 3214 in $S_n - e'$. Then $P + e$ is a cycle of length 24 containing e in $S_4 - e'$. Let ℓ be any even integer with $6 \leqslant \ell \leqslant 22$. To complete the proof of the lemma, we need to construct a set of cycles of length ℓ such that the intersection of their edge sets is e. We give the construction detail in Appendix. □

Lemma 4. *For any edge e and an edge set F with $|F| = n-3$ and $n \geqslant 5$, there exists a subgraph $S^{i:j}_{n-1} = (V^{i:j}, E^{i:j})$ such that $e \in E^{i:j}$ and $|E^{i:k} \cap F| \leqslant n-4$ for all $1 \leqslant k \leqslant n$.*

Proof. Without loss of generality, we assume $e = (a_1a_2 \cdots a_i \cdots a_n, a_ia_2 \cdots a_1 \cdots a_n)$. Consider the $n-2$ subgraphs $S^{2:a_2}_{n-1}, S^{3:a_3}_{n-1}, \ldots, S^{i-1:a_{i-1}}_{n-1}, S^{i+1:a_{i+1}}_{n-1}, \ldots, S^{n:a_n}_{n-1}$. Obviously, $e \in \cap_{k \neq 1,i} E^{k:a_k}$. Then for each j where $2 \leqslant j \leqslant n$, $j \neq i$, $\{V^{j:1}, V^{j:2}, \ldots, V^{j:n}\}$ is a partition of $V(S_n)$. If none of these partitions satisfies Lemma 4 then the endpoints of all the edges in F have the same digit value in bit position from 2 to n excluding bit position i. This condition can only be satisfied when there is only one edge in the set F. Since $|F| = n-3 > 1$ for $n \geqslant 5$, we get a contradiction. So Lemma 4 holds. □

In [8], Li proposed the following algorithm for constructing $(n-2)$-disjoint paths from which they proved Lemma 6.

Algorithm 5
Input m, n, where $n \geqslant 4$ and $3 \leqslant m \leqslant n$.
Output $(n-2)$ disjoint paths.

1. Select arbitrary $(n-2)$ vertices, say $a^1, a^2, \ldots, a^{n-2}$ from $V(S^m_{n-1})$ with the first digits equal to 1 //Note that the last bits of $a^1, a^1, \ldots, a^{n-2}$ are m.
2. For $i \leftarrow 1$ to $(n-2)$
3. $\quad P_i \leftarrow \langle a^i \rangle$For $j \leftarrow 2$ to m
4. $\qquad v \leftarrow lastVertex(P_i)$ //Suppose that $v = v_1v_2 \cdots v_{n-1}v_n$.
5. $\qquad\quad$ //$lastVertex(P_i)$ returns the last vertex in P_i.
6. $\qquad$ Append $v_nv_2 \cdots v_{n-1}v_1$ to P_i
7. $\qquad$ Append $v_lv_2 \cdots v_{l-1}v_nv_{l+1} \cdots v_{n-1}v_1$ to P_i where $v_l = j$
8. $\quad$ Append $v_nv_2 \cdots v_{n-1}v_1$ to P_i, where v is the last vertex in P_i
9. Output $P_1, P_2, \ldots, P_{n-2}$.

Lemma 6 (Li [8]). *There are $(n-2)$ disjoint paths of length $(2m-1)$ crossing $S^1_{n-1}, S^2_{n-1}, \ldots, S^m_{n-1}$ such that the endpoints of these paths are in S^m_{n-1} for $n \geqslant 3$ and $3 \leqslant m \leqslant n$. The endpoints of these paths are adjacent for $m = 3, 4$.*

Let $e_1 = (x^1, y^1)$, $e_2 = (x^2, y^2), \ldots, e_{n-2} = (x^{n-2}, y^{n-2})$ be a set of $(1,2,m)$-edges in S^1_{n-1} for $n \geqslant 3$ with $x^1_1 = x^2_1 = \cdots = x^{n-2}_1 = m$ and $y^1_1 = y^2_1 = \cdots = y^{n-2}_1 = 2$. In the step 1 of Algorithm 5, if we choose $a_i = x^i_n x^i_2 \cdots x^i_{n-1} x^i_1$ when $x^i = x^i_1 x^i_2 \cdots x^i_{n-1} x^i_n$ for $1 \leqslant i \leqslant n-2$, then we can get $n-2$ disjoint paths $P_1, P_2, \ldots, P_{n-2}$ of length $(2m-1)$ crossing S^1_{n-1}, $S^2_{n-1}, \ldots, S^m_{n-1}$ such that the endpoints of these paths are in S^m_{n-1} for $n \geqslant 3$ and $3 \leqslant m \leqslant n$ by the Algorithm 5. Furthermore, we have $E(P_i) \cap E(S^1_{n-1}) = e_i$ for $1 \leqslant i \leqslant n-2$.

Since S_n is vertex-symmetric and edge-symmetric, by the construction of above paths, we can get the following lemma easily.

Lemma 7. *Let* $\{e_1, e_2, \ldots, e_{n-2}\}$ *be a set of* (i_1, i_2, i_m)*-edges in* $S^{i_1}_{n-1}$. *Then there exists* $n-2$ *disjoint paths* $P_1, P_2, \ldots, P_{n-2}$ *of length* $2m-1$ *crossing* $S^{i_1}_{n-1}, S^{i_2}_{n-1}, \ldots, S^{i_m}_{n-1}$ *such that* $e_i \in P_i$ *and the endpoints of these paths are in* $S^{i_m}_{n-1}$ *for* $n \geqslant 3$ *and* $3 \leqslant m \leqslant n$. *The endpoints of these paths are adjacent for* $m = 3, 4$.

The above lemma guarantees the existence of a fault-free path when the number of fault edges in S_n does not exceed $(n-3)$.

Lemma 8. *Let* $F \subseteq E(S_n)$ *with* $|F| \leqslant n-3$ *for* $n \geqslant 3$ *and* $\{e_1, e_2, \ldots, e_{n-2}\}$ *be a set of* (i_1, i_2, i_m)*-edges in* $S^{i_1}_{n-1}$. *Then there exists a path* P *in* $S_n - F$ *of length* $(2m-1)$ *crossing* $S^{i_1}_{n-1}, S^{i_2}_{n-1}, \ldots, S^{i_m}_{n-1}$ *such that the endpoints of* P *are in* $S^{i_m}_{n-1}$ *for* $n \geqslant 3$ *and* $3 \leqslant m \leqslant n$. *Furthermore,* $|E(P) \cap \{e_1, e_2, \ldots, e_{n-2}\}| = 1$. *The endpoints of these paths are adjacent for* $m = 3, 4$.

3. Edge-pancyclicity of star graphs with edge-fault

In this section, we will give the proof of our main result.

Theorem 9. *For any edge subset* F *of* S_n *with* $|F| \leqslant n-3$ *and any edge* $e \in S_n - F$, *there exists a cycle of even length from* 6 *to* $n!$ *in* $S_n - F$ *containing* e *provided* $n \geqslant 3$.

Proof. We prove the theorem by induction on $n \geqslant 3$. For $n = 3$, the star graph S_3 is a cycle of length six. Since $|F| \leqslant n-3 = 0$, the theorem holds for $n = 3$.

For $n = 4$, the theorem holds by Lemma 3.

Assume now that the theorem is true for all integer k, $3 \leqslant k < n$. Let F be any edge subset of S_n with $|F| \leqslant n-3$, e be any edge of $S_n - F$ and ℓ be any even integer with $6 \leqslant \ell \leqslant n!$, where n is an integer no less than 5. Since S_n is vertex-symmetric and edge-symmetric, we assume that $e = (a_1 a_2 \cdots a_{i-1} a_i a_{i+1} \cdots a_n, a_i a_2 \cdots a_{i-1} a_1 a_{i+1} \cdots a_n)$ for $2 \leqslant i < n$ and $|F \cap E(S^m_{n-1})| \leqslant n-4$ for $1 \leqslant m \leqslant n$ by Lemma 6. Obviously, $e \in E(S^{a_n}_{n-1})$. To complete the proof of the theorem, we need to show that there exists a cycle of length ℓ in $S_n - F$ containing e.

Case 1. $6 \leqslant \ell \leqslant (n-1)!$ By the induction hypothesis, there exists a cycle C of length ℓ in $S^{a_n}_{n-1}$ containing e. Specially, we use C_1 and C_2 to denote the cycles of length $(n-1)! - 2$ and $(n-1)!$, respectively.

Case 2. $(n-1)! + 2 \leqslant \ell \leqslant 3(n-1)!$ In this case, we can write $\ell = \ell_1 + \ell_2 + \ell_3$ where ℓ_1, ℓ_2 and ℓ_3 satisfy the following conditions:

$$
\begin{aligned}
&\ell_1 = (n-1)! - 2 && \text{or } (n-1)! \\
&\ell_2 = 2 && \text{or } 6 \leqslant \ell_2 \leqslant (n-1)! \\
&\ell_3 = 2 && \text{or } 6 \leqslant \ell_3 \leqslant (n-1)!
\end{aligned}
$$

We consider the cycle C' in $S^{a_n}_{n-1}$ where $C' = C_1$ if $\ell_1 = (n-1)! - 2$, and $C' = C_2$ otherwise. Since $\left\lceil \frac{2 \times ((n-1)! - 2)}{(n-1)(n-2)} \right\rceil \geqslant n-1$ for $n \geqslant 5$, then there are at least $n-2$ edges in a set of (a_n, x_j, x_k)-edges E where $e \notin E$ and $1 \leqslant x_j$, $x_k \leqslant n$, $x_j \neq a_n$, $x_k \neq a_n$. By Lemma 8, there exists a cycle C^* of length six crossing $S^{a_n}_{n-1}$, $S^{x_j}_{n-1}$ and $S^{x_k}_{n-1}$ in $S_n - F$ containing an edge e' where $e' \in E$, $E(C^*) \cap E(C') = e'$, $E(C^*) \cap E(S^{x_j}_{n-1}) = e_j$ and $E(C^*) \cap E(S^{x_k}_{n-1}) = e_k$. By the induction hypothesis, there exists a cycle C_k of length ℓ_2 in $S^{x_j}_{n-1} - F$ containing e_j if $\ell_2 \geqslant 6$ and exists a cycle of length ℓ_3 in $S^{x_k}_{n-1} - F$ containing e_k if $\ell_3 \geqslant 6$.

In the cycle C^*, we replace the edge e' by the cycle $C' - e'$. Replace the edge e_j by the cycle $C_j - e_j$ if $\ell_j \geqslant 6$. Replace the edge e_k by the cycle $C_k - e_k$ if $\ell_k \geqslant 6$. Then, we get the cycle of length ℓ in $S_n - F$ containing e.

Case 3. $3(n-1)! + 2 \leqslant \ell \leqslant n!$ In this case, we can write $\ell = \ell_1 + \ell_2 + \ell_3 + \cdots + \ell_n$ where $\ell_1 \geqslant \ell_2 \geqslant \ell_3 \geqslant \cdots \geqslant \ell_n$ and the following restrictions should be satisfied:

$$\begin{aligned}
&\ell_1 = (n-1)! - 2 && \text{or } (n-1)! \\
&\ell_2 = (n-1)! - 2 && \text{or } (n-1)! \\
&\ell_3 = 2 && \text{or } 6 \leqslant \ell_3 \leqslant (n-1)! \\
&\ell_4 = 0 && \text{or } 2 \text{ or} 6 \leqslant \ell_3 \leqslant (n-1)! \\
&\quad \cdots \\
&\ell_n = 0 \text{ or } 2 && \text{or } 6 \leqslant \ell_n \leqslant (n-1)!
\end{aligned}$$

Let $M = \{\ell_i : \ell_i \neq 0,\ 1 \leqslant i \leqslant n\}$ and $m = |M|$. We consider the cycle C' in $S_{n-1}^{a_n}$ where $C' = C_1$ if $\ell_1 = (n-1)! - 2$, otherwise $C' = C_2$. Since $\left\lceil \frac{2\times((n-1)!-2)}{(n-1)(n-2)} \right\rceil > n-1$ for $n \geqslant 5$, then there are at least $n-2$ edges in a set of (a_n, x_j, x_k)-edges E where $e \notin E$ and $1 \leqslant x_j, x_k \leqslant n$, $x_j \neq a_n$, $x_k \neq a_n$. By Lemma 8, there exists a path P in $S_n - F$ of length $(2m-1)$ crossing $S_{n-1}^{x_k}, S_{n-1}^{a_n}, S_{n-1}^{x_j}, S_{n-1}^{b_4}, \ldots, S_{n-1}^{b_m}$ such that the endpoints of path P are in $S_{n-1}^{x_k}$ for $n \geqslant 5$ and $4 \leqslant m \leqslant n$. Furthermore, the path P contains an edge $e' \in E$ where $E(P) \cap E(S_{n-1}^{a_n}) = e'$ and $E(P) \cap E(S_{n-1}^{x_j}) = e''$ and $E(P) \cap E(S_{n-1}^{b_i}) = e_i$ for $4 \leqslant i \leqslant m$. By Theorem 2, there exists a Hamiltonian path P_k of length $(n-1)! - 1$ in $S_{n-1}^{x_k} - F$ joining the endpoints of P. And by the induction hypothesis, there exists a cycle C'' of length ℓ_3 in $S_{n-1}^{x_j} - F$ containing e'' if $\ell_3 \geqslant 6$ and a cycle C_i of length ℓ_i in $S_{n-1}^{b_i} - F$ containing e_i for $4 \leqslant i \leqslant m$ if $\ell_i \geqslant 6$.

We now produce a cycle of length ℓ in $S_n - F$ containing e by substituting some edges in path P as follows: Joint the endpoints of P by path P_k, that makes a cycle and then replace edge $e' \in P$ by the path $C' - e'$. Next replace the edge e'' by the cycle $C'' - e''$ if $\ell_3 \geqslant 6$ and replace the edge e_i by the cycle $C_i - e_i$ for $4 \leqslant i \leqslant m$ if $\ell_i \geqslant 6$. As a result we obtain the cycle of desired length.

Then the theorem holds. □

4. Conclusion

In this paper, we show that for any edge subset F of S_n with $|F| \leqslant n-3$ and any edge $e \in S_n - F$, there exists a cycle of even length from 6 to $n!$ in $S_n - F$ containing e provided $n \geqslant 3$. If the $(n-2)$ faulty edges are incident with the same vertex, then the faulty S_n contains no cycle of length $n!$. And since the length of shortest cycle in S_n is six, then our result is optimal.

Appendix. Construction details of desired cycles

If $\ell = 6$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 2134, 1234\rangle$ and $\langle 1234, 3214, 4213, 1243, 3241, 4231, 1234\rangle$ are the desired cycles. See Fig. 2.

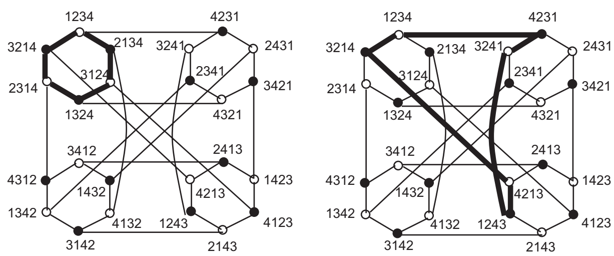

Fig. 2. The desired cycles of length 6.

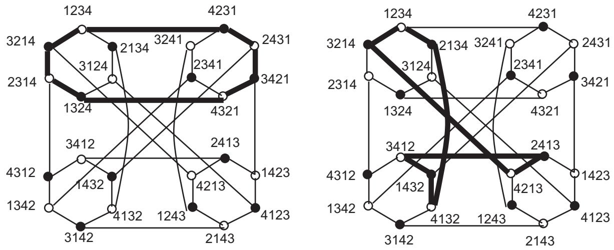

Fig. 3. The desired cycles of length 8.

If $\ell = 8$, the cycles $\langle 1234, 3214, 2314, 1324, 4321, 3421, 2431, 4231, 1234 \rangle$ and $\langle 1234, 3214, 4213, 2413, 3412, 1432, 4132, 2134, 1234 \rangle$ are the desired cycles. See Fig. 3.

If $\ell = 10$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 4123, 1423, 3421, 2431, 4231, 1234 \rangle$ and $\langle 1234, 3214, 4213, 1243, 3241, 2341, 1342, 3142, 4132, 2134, 1234 \rangle$ are the desired cycles. See Fig. 4.

If $\ell = 12$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 4123, 1423, 3421, 4321, 2341, 3241, 4231, 1234 \rangle$ and $\langle 1234, 3214, 4213, 1243, 2143, 3142, 1342, 4312, 3412, 1432, 4132, 2134, 1234 \rangle$ are the desired cycles. See Fig. 5.

If $\ell = 14$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 4123, 2143, 1243, 4213, 2413, 3412, 1432, 4132, 2134, 1234 \rangle$, $\langle 1234, 3214, 2314, 4312, 1342, 3142, 2143, 4123, 1423, 3421, 4321, 2341, 3241, 4231, 1234 \rangle$ and $\langle 1234, 3214, 4213, 1243, 2143, 3142, 4132, 1432, 2431, 3421, 4321, 2341, 3241, 4231, 1234 \rangle$ are the desired cycles. See Fig. 6.

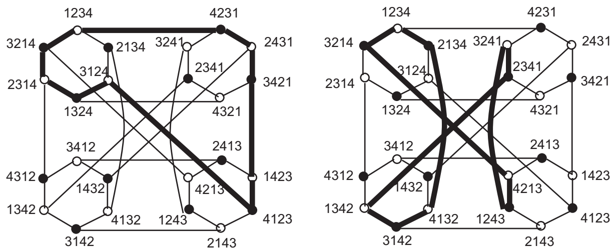

Fig. 4. The desired cycles of length 10.

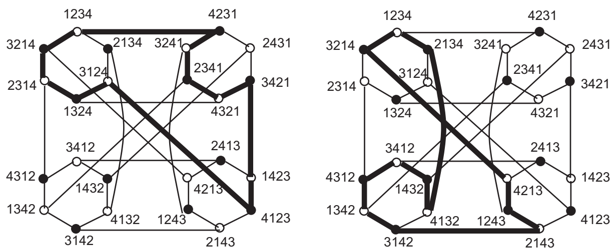

Fig. 5. The desired cycles of length 12.

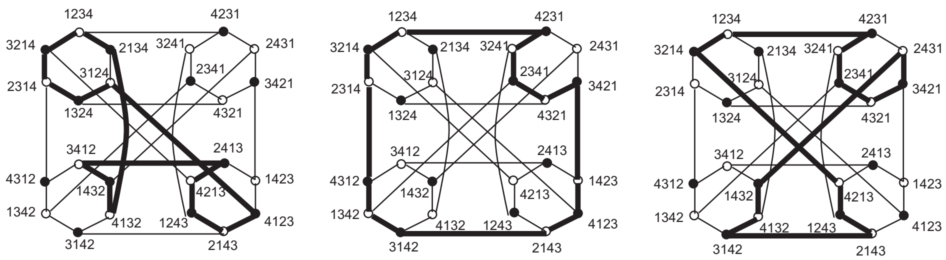

Fig. 6. The desired cycles of length 14.

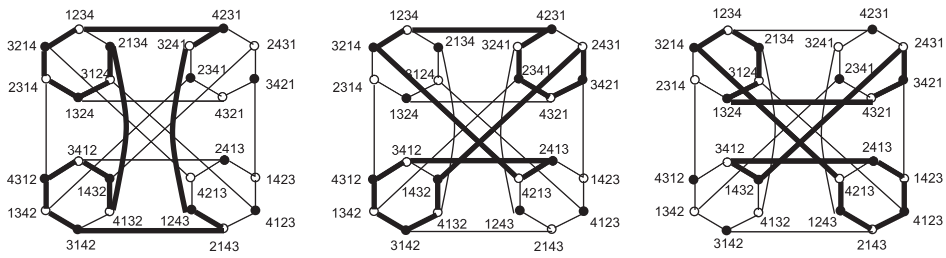

Fig. 7. The desired cycles of length 16.

If $\ell = 16$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 2134, 4132, 1432, 3412, 4312, 1342, 3142, 2143, 1243, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 4213, 2413, 3412, 4312, 1342, 3142, 4132, 1432, 2431, 3421, 4321, 2341, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 4213, 1243, 2143, 4123, 1423, 2413, 3412, 1432, 2431, 3421, 4321, 1324, 3124, 2134, 1234\rangle$ are the desired cycles. See Fig. 7.

If $\ell = 18$, the cycles $\langle 1234, 3214, 2314, 4312, 1342, 3142, 2143, 4123, 1423, 2413, 4213, 1243, 3241, 2341, 4321, 3421, 2431, 4231, 1234\rangle$, $\langle 1234, 3214, 4213, 2413, 1423, 3421, 4321, 2341, 1342, 3142, 4132, 1432, 3412, 4312, 2314, 1324, 3124, 2134, 1234\rangle$ and $\langle 1234, 3214, 2314, 1324, 3124, 2134, 4132, 3142, 2143, 1243, 3241, 2341, 1342, 4312, 3412, 1432, 2431, 4231, 1234\rangle$ are the desired cycles. See Fig. 8.

If $\ell = 20$, the cycles $\langle 1234, 3214, 4213, 1243, 2143, 4123, 1423, 2413, 3412, 4312, 1342, 3142, 4132, 1432, 2431, 3421, 4321, 2341, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 2314, 1324, 3124, 2134, 4132, 1432, 3412, 4312, 1342, 3142, 2143, 4123, 1423, 2413, 4213, 1243, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 4213, 2413, 1423, 4123, 2143, 1243, 3241, 4231, 2431, 3421, 4321, 2341, 1342, 4312, 2314, 1324, 3124, 2134, 1234\rangle$ and $\langle 1234, 3214, 2314, 1324, 3124, 2134, 4132, 1432, 3412, 4312, 1342, 3142, 2143, 1243, 3241, 2341, 4321, 3421, 2431, 4231, 1234\rangle$ are the desired cycles. See Fig. 9.

If $\ell = 22$, the cycles $\langle 1234, 3214, 2314, 1324, 3124, 4123, 2143, 1243, 4213, 2413, 3412, 4312, 1342, 3142, 4132, 1432, 2431, 3421, 4321, 2341, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 2314, 1324, 3124, 2134, 4132, 3142, 1342, 4312, 3412, 2413, 4213, 1243, 2143, 4123, 1423, 3421, 4321, 2341, 3241, 4231, 1234\rangle$, $\langle 1234, 3214, 4213, 2413, 1423, 4123, 2143, 3142, 4132, 1432, 3412, 4312, 1342, 2341, 3241, 4231, 2431, 3421, 4321, 1324, 3124, 2134, 1234\rangle$, $\langle 1234, 3214, 4213, 1243, 2143, 4123, 1423, 2413, 3412, 1432, 4132, 3142, 1342, 2341, 3241, 4231, 2431, 3421, 4321, 1324, 3124, 2134,$

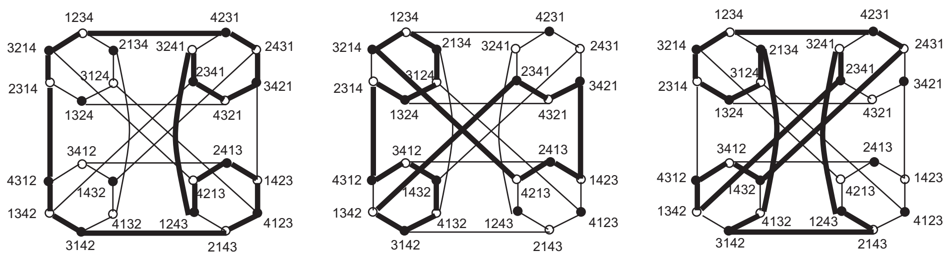

Fig. 8. The desired cycles of length 18.

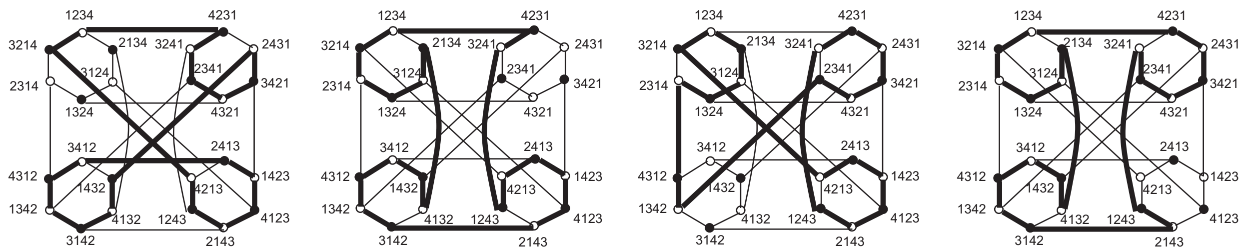

Fig. 9. The desired cycles of length 20.

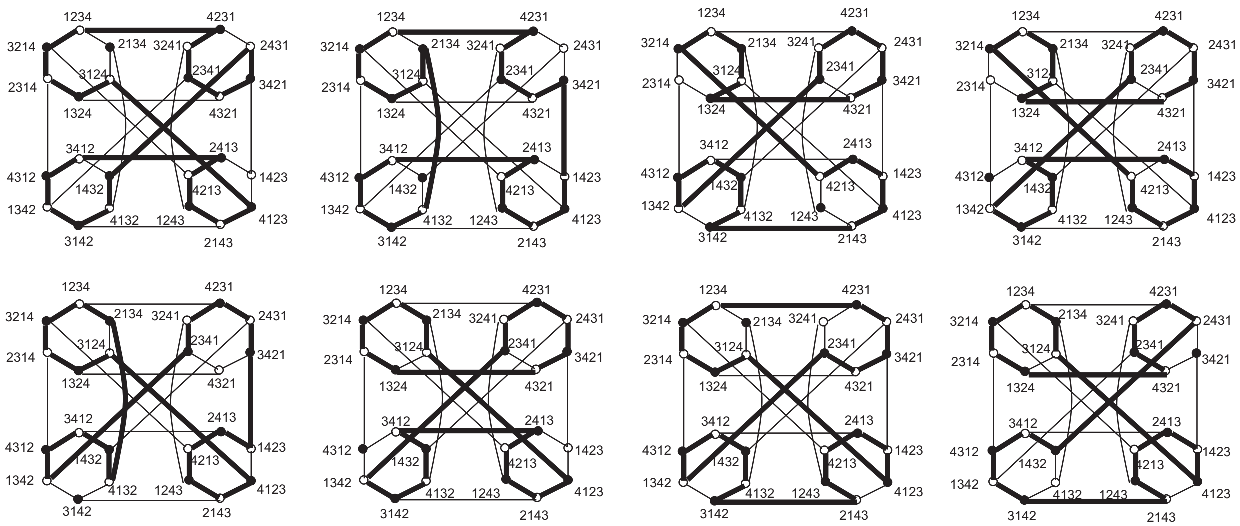

Fig. 10. The desired cycles of length 22.

1234⟩, ⟨1234, 3214, 2314, 1324, 3124, 4123, 2143, 1243, 4213, 2413, 1423, 3421, 2431, 4231, 3241, 2341, 1342, 4312, 3412, 1432, 4132, 2134, 1234⟩, ⟨1234, 3214, 2314, 1324, 4321, 3421, 2431, 4231, 3241, 2341, 1342, 3142, 4132, 1432, 3412, 2413, 4213, 1243, 2143, 4123, 3124, 2134, 1234⟩, ⟨1234, 3214, 2314, 1324, 3124, 4123, 1423, 2413, 4213, 1243, 2143, 3142, 4132, 1432, 3412, 4312, 1342, 2341, 4321, 3421, 2431, 4231, 1234⟩ and ⟨1234, 3214, 2314, 1324, 4321, 2341, 3241, 4231, 2431, 1432, 3412, 4312, 1342, 3142, 2143, 1243, 4213, 2413, 1423, 4123, 3124, 2134, 1234⟩ are the desired cycles. See Fig. 10.

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