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Mixed integer linear programming problem for personnel multi-day shift scheduling: A case study in an Iran hospital

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Abstract The application of mathematical and systematic models and approaches to health is on the rise. In this study, a mixed-integer linear programming problem with cost (staff payroll) objective function is proposed to determine homogeneous personnel shift scheduling so that the constraints of the number of personnel per shift are met. The purpose of this study is to optimize the scheduling of personnel with the same skills to minimize variable hospital costs. In a hospital, homogeneous personnel might be skilled nurses who can work in the same departments or service staff of the departments. First, a mathematical model is proposed for scheduling as a linear problem and solved using Lingo software. By analyzing the results for a personnel working time period in Pasargad Hospital in Tehran (Iran), it is found that while optimizing the utilization of the skilled staff within departments, the variable costs (payroll) related to the department for that specified period (month), which includes overtime, overnight and holidays, are reduced by around 10 percent.

1. Introduction

The problem of personnel scheduling involves the efficient allocation of work activities to employees while complying with union rules and minimizing current costs. These problems are applicable to many work environments, such as banks, hospitals, restaurants, and stores. [1] The variable costs of health care in developing countries are increasing constantly,

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and hospitals and health centers are looking for cost-cutting strategies. More than 50 percent of the variable cost of hospitals is the payroll of the personnel. Therefore, finding scientific approaches that can reduce this cost is a reasonable way to reduce costs and increase profits. Over the years, one of the most important concerns of hospital management has been staff utilization optimization and personnel scheduling decisions. The consequences of inappropriate staffing can have a negative impact on hospital performance, patient experience, and staff satisfaction [2]. The number of personnel required for service sections like healthcare and police services is predetermined and can be categorized into sustainability-based planning [3,4]. In general, organizations can be classified into 4 different categories when it comes to personnel scheduling. Besides sustainability-based, the second category is demand fluctuation-based personnel planning, which is applicable to warehouses, distribution centers, restaurants, and postal services. The third one is mobility-based planning and is applicable to transportation companies such as airlines and bus companies. The fourth and final class relates to companies that are assigned different projects and they must divide these projects between personnel. This category is called project-based planning [5].

Mathematical models of the problem of scheduling work shifts are presented in some articles. Henderson and Bery proposed three heuristic algorithms to identify subsets of all possible shift patterns [6]. Monroe developed the problem of work shift schedules for consecutive vacation days [7]. Tibriwala et al. presented a simple heuristic method for selecting a work shift pattern with two consecutive days off [8]. Bailey then proposed an integrated model for the scheduling problem by considering holidays and scheduling shifts. In his model, several working shift patterns are assumed to be assigned to personnel at a minimum cost for all shifts [9].

The personnel scheduling problem can also be categorized into two single-section and multi-section categories. In single-section planning, personnel of a section or department within an organization are scheduled independently of the needs of other departments [10]. But in the multi-section problem, it is possible to exchange personnel between departments and units. Bard and Van [11] and Leggow et al. [12] presented the task allocation problem for single-section job shifts. In recent years, Wennberg et al. [13] and DeBrooker et al. [14] have reviewed personnel scheduling problem studies. Bogild and Jepson [15] examined the impact of several consecutive shifts, long weekends, and two different shifts (day-evening and day-night shifts) on heart disease biomarkers.

The problem of personnel scheduling (nurses) is the allocation of several nurses to a number of shifts to meet the hospital's demands. The objective of this problem is to minimize overall hospital costs and maximize preferences regarding legislative laws and hospital standards. M'Hallah and Alkhabbaz [16] conducted a real case study on a health care center in Kuwait. They formulated a mixed-integer programming problem with the goal of minimizing the outsourcing of nurses, and their preferences. Lim et al. [17] proposed an effective solution approach for multi-skilled nurse scheduling with lunch break assignment in operating. Their approach consists of two basic optimization problems for scheduling nurses in the medical center. The first model is a multi-objective mixed-integer problem of allocating nurses to upcoming surgical patients based on their skill levels. The second model is a nurse lunch break assignment model. Eladoli et al. [18] proposed a mathematical model for scheduling nurses' shifts based on the idea of a multi-state network flow model. The model presented was implemented for a real case study in a hospital in Egypt. The results of their study showed that this mathematical model is more equitable compared to the manually scheduled shifts by the manager, and reduces the overall overtime cost by approximately 36 percent. Koribal et al. [19] have developed an idealized scheduling method for scheduling nurses' shifts for 4 different sections. That same year, Dahman et al. [20] proposed a multi-section, multi-day work shifts scheduling problem for a heterogeneous multi-skilled workforce in which personnel moves between departments with some constraints.

Based on previous researches, one of the most important issues and challenges hospital managers are dealing with is optimizing staff schedules. The purpose of this study is to optimally schedule homogenous (with the same skills) personnel's shifts, taking into account the days off and the number of staff required per shift to comply with labor laws and minimize the current costs of staff payroll. In this proposed mathematical model, a comprehensive mixed-integer linear programming problem is presented for scheduling normal hospital shifts for homogeneous personnel. The differences between this study and the previous researches on personnel scheduling are: (a) Mathematical model is formulated regarding the legislative rules, (b) Off days can be determined in the model, (c) Working shifts are different and varies by time, (d) Overtime days and shifts of each staff are specified, (f) Excess overtime, overnight and holiday costs are considered, (g) Personnel satisfaction is also increased by taking into account resting times and proper working hours, overtime and attention to the maximum capacity of each person. Finally, by solving the proposed model and implementing the results, the current costs can be reduced.

This model is presented in a general form approximately, but at the same time, we considered a case study in an Iran hospital to explain the problem and ease of understanding and solving. However, this explanation does not mean that the presentation is for a specific case because any researcher can change or name the parameters of the problem (for example, minimum and maximum working time, working time, maximum and minimum day in each shift, number of shifts like morning, evening and night, etc.) based on the problem under their circumstances. Meanwhile, the researchers have to notice their parameters and circumstances and adapt their problems with this study. In this research, we have considered multi-shift programming, which includes work in the morning, evening, and overnight, so that the problem is closer to the real world. Staffs' effectiveness can also increase by considering the constraints of the model we proposed, for instance, one worker cannot work 24 h a day. On the other hand, we have assumed that the payroll of shifts could be different. For example, working night shifts is normally much harder than working morning shifts. Besides, this model can also help planners and managers to determine the appropriate number of staff for their scheduling program. Although there are no binary variables for hiring and firing staff, and we just schedule the existing staff in the proposed model, but we can perform a sensitivity analysis on the number of staff, and then the appropriate number is obtained based on its results. Finally, we tried to provide a personnel scheduling model for the real work environment with existing laws that apply in many countries. Some of these rules are: per shift maximum overtime is specified, overnight payroll costs are more than morning payroll costs, etc.

2. Mathematical modeling

The problem addressed in this study is to schedule shifts for the number of hospital personnel required. Prior to the mathematical modeling of the proposed problem, it is necessary to state the assumptions used.

- The total number of personnel to get scheduled is determined.

- All staff are homogeneous, meaning they have relatively same skills.

- The hourly wages of any staff member (new or experienced) are known.

- The total number of staff required per shifts of each day is determined.

- There are four types of shifts per day:

- Morning (D) from 7 to 13, 6 h;
- Evening (G) from 13 to 19, 6 h;
- Morning and evening (L) from 7 to 19, 12 h;
- Overnight (N) from 19 to 7, 12 h.

- Personnel who work 12-hour shifts must rest the following day, i.e. 12 h of work and 24 h of vacation.

- The overtime, holidays and overnight payroll costs is determined.

- The overnight working hours is 10 pm to 7 am.

- Each staff member must work 180 h per month and can be assigned to one shift from the above four types.

- The number of workdays in each period is 30 days and the holidays are pre-determined from the calendar.

- Total and per shift maximum overtime is specified.

- No staff can work 24 h a day.

The symbols used in this linear programming problem are as follows. The indices i and j represent i^{th} staff and j^{th} workday.

m: Total number of staff

r: Total number of workdays

T_j: Set of off days

S_i: Monthly payroll of staff i , including base pay and all other benefits

C_i: Payroll of 1-hour work of staff i, yields by dividing the base pay minimum working time of each staff (S_i/W)

W: Minimum working time of each staff

V: Maximum working time of each staff

F: Maximum day that L-type staff can work in the morning and/or evening shifts as overtime

B: Maximum day that D/G-type staff can work in the morning and/or evening shifts as overtime

x: Maximum day that D/G-type staff can work in the night shifts as overtime

f: Maximum day that N-type staff can work in the morning and/or evening shifts as overtime

E_j: Minimum number of staff needed in the morning for j day

O_j: Minimum number of staff needed in the evening for j day

P_j: Minimum number of staff needed in the night for j day

α: Percentage added to base salary for working on overtime or holiday

β: Percentage added to base salary for working on overnight

Decision variables:

D_i: Binary variable determines if staff i works on the morning shift or not

G_i: Binary variable determines if staff i works on the evening shift or not

L_i: Binary variable determines if staff i works on the morning and evening shift or not

N_i: Binary variable determines if staff i works on the night shift or not

d_{ij}: Binary variable determines if staff i works on the morning shift of day j or not

g_{ij}: Binary variable determines if staff i works on the evening shift of day j or not

l_{ij}: Binary variable determines if staff i works on the morning and evening shift of day j or not

n_{ij}: Binary variable determines if staff i works on the night shift of day j or not

U_i: Total working hours of staff i

t_{ij}: Total off-day working hours of staff i on day j

ng_{ij}: Binary variable determines if G-type staff i works on the night shift of day j or not

gn_{ij}: Binary variable determines if N-type staff i works on the evening shift of day j or not

gl_{ij}: Binary variable determines if L-type staff i works on the evening shift of day j or not

gd_{ij}: Binary variable determines if D-type staff i works on the evening shift of day j or not

nd_{ij}: Binary variable determines if D-type staff i works on the night shift of day j or not

dn_{ij}: Binary variable determines if N-type staff i works on the morning shift of day j or not

dl_{ij}: Binary variable determines if L-type staff i works on the morning shift of day j or not

dg_{ij}: Binary variable determines if G-type staff i works on the morning shift of day j or not

The proposed mathematical model with respect to the mentioned variables is as follows:

$$MinZ = \sum_{j=1}^{m}\left\{\left(\sum_{i=1}^{m} S_i(D_i + G_i + L_i + N_i)\right) + (\alpha C_i(U_i - W)) + \left(\alpha C_i \sum_{j=1}^{r} t_{ij}\right) + \beta C_i \sum_{j=1}^{r}\left(nd_{ij} + ng_{ij} + n_{ij}\right)\right\} \tag{1}$$

S.t:

$$D_i + G_i + L_i + N_i = 1; \forall i \tag{2}$$

$$d_{ij} \geq 26D_i; \forall i.j \tag{3}$$

$$d_{ij} \leq D_i; \forall i.j \tag{4}$$

$$g_{ij} \geq 26G_i; \forall i.j \tag{5}$$

$$g_{ij} \leq G_i; \forall i.j \tag{6}$$

$$l_{ij} \geq 15L_i; \forall i.j \tag{7}$$

$$l_{ij} \leq L_i; \forall i.j \tag{8}$$

$$l_{ij} + l_{i(j+1)} \leq L_i; \forall i.j = 1.2 \cdots r \tag{9}$$

$$n_{ij} \geq 15 N_i; \forall i.j \tag{10}$$

$$n_{ij} \leq N_i; \forall i.j \tag{11}$$

$$n_{ij} + n_{i(j+1)} \leq N_i; \forall i.j = 1 \cdots r \tag{12}$$

$$ng_{ij} \leq G_i x; \forall i.j \tag{13}$$

$$gn_{ij} \leq N_i f; \forall i.j \tag{14}$$

$$n_{ij} + gn_{i(j+1)} \leq N_i; \forall i.j = 1 \cdots r \tag{15}$$

$$nd_{ij} \leq D_i x; \forall i.j \tag{16}$$

$$dn_{ij} \leq N_i f; \forall i.j \tag{17}$$

$$n_{ij} + dn_{ij} \leq N_i; \forall i.j \tag{18}$$

$$gd_{ij} \leq D_i B; \forall i.j \tag{19}$$

$$dg_{ij} \leq G_i B; \forall i.j \tag{20}$$

$$dl_{ij} \leq L_i F; \forall i.j \tag{21}$$

$$dl_{ij} + l_{ij} \leq L_i; \forall i.j \tag{22}$$

$$gl_{ij} \leq L_i F; \forall i.j \tag{23}$$

$$gl_{ij} + l_{ij} \leq L_i; \forall i.j \tag{24}$$

$$\sum_{i=1}^{m} \left(d_{ij} + dn_{ij} + dl_{ij} + dg_{ij} + l_{ij} \right) \geq E_j; \forall j \tag{25}$$

$$\sum_{i=1}^{m} \left(g_{ij} + gn_{ij} + gl_{ij} + gd_{ij} + l_{ij} \right) \geq O_j; \forall j \tag{26}$$

$$\sum_{i=1}^{m} \left(n_{ij} + ng_{ij} + nd_{ij} \right) \geq P_j; \forall j \tag{27}$$

$$U_i = \sum_{j=1}^{r} \left\{ 6\left(d_{ij} + dn_{ij} + dl_{ij} + dg_{ij} + g_{ij} + gn_{ij} + gl_{ij} + gd_{ij} \right) + 12\left(l_{ij} + n_{ij} + ng_{ij} + nd_{ij} \right) \right\}; \forall i \tag{28}$$

$$W \leq U_i \leq V; \forall i \tag{29}$$

$$t_{ij} = 6\left(d_{ij} + dn_{ij} + dl_{ij} + dg_{ij} + g_{ij} + gn_{ij} + gl_{ij} + gd_{ij} \right) + 12\left(ng_{ij} + nd_{ij} \right); \forall i \in m.j \in T_j \tag{30}$$

$$D_i.G_i.L_i.N_i \in \{0.1\}; \forall i \tag{31}$$

$$d_{ij}.g_{ij}.l_{ij}.n_{ij}.ng_{ij}.nd_{ij}.gn_{ij}.dn_{ij}. \tag{32}$$

$$gl_{ij}.dl_{ij}.gd_{ij}.dg_{ij} \in \{0.1\}; \forall i.j$$

$$U_i.t_{ij} \geq 0; \forall i.j \tag{33}$$

The objective function (1) is regarding minimization of the total variable cost of payrolls, including base pay, benefits, overtime, and overnight payments. Constraint (2) ensures that each staff member is assigned exactly one shift. Constraint (3) ensures that staff with morning (D) shifts must work at least 26 days in a month for this shift to complete 180 h of work, which is the minimum working time of each staff (W). Constraint (4) ensures that staff who selected for morning shifts work at morning shifts. Constraint (5) ensures that staff with evening (G) shifts must work at least 26 days in a month for this shift to complete 180 h of work. Constraint (6) ensures that staff who selected for evening shifts work at evening shifts. Constraint (7) ensures that staff with morning and evening (L) shifts must work at least 15 days in a month for this shift to complete 180 h of work. Constraint (8) ensures that staff who selected for morning and evening shifts work at this shift. Constraint (9) ensures that staff who worked at morning and evening shifts be off the next day to comply with the 12-hours work 24-hours off rule. Constraint (10) ensures that staff with night (N) shifts must work at least 15 days in a month for this shift to complete 180 h of work. Constraint (11) ensures that staff who selected for night shifts work at this shift. Constraint (12) ensures that staff who worked at night shifts be off the next day to comply with the 12-hours work 24-hours off rule.

Constraint (13) states that staff with evening shifts may work up to × night(s) as overtime. Constraint (14) states that staff with night shift may work up to f evening shift(s) as overtime. Constraint (15) ensures that the overtime evening shift for the night-shift staff must be the same shift before that night, to ensure staff satisfaction. Constraint (16) states that staff with morning shift may work up to × night(s) as overtime. Constraint (17) states that staff with night shift may work up to f morning shift(s) as overtime. Constraint (18) ensures that the overtime morning shift for the night-shift staff must be the same shift after that night, to ensure staff satisfaction. Constraint (19) states that staff with morning shifts may work up to B evening shift(s) as overtime. Constraint (20) states that staff with evening shifts may work up to B morning shift(s) as overtime. Constraint (21) states that staff with morning and evening (L) shifts may work up to F days of morning shift(s) as overtime. Constraint (22) ensures that staff with morning and evening (L) shift just work at morning shift as overtime in the days which are not assigned to them as L shifts. Constraint (23) states that staff with morning and evening (L) shifts may work up to F days of evening shift(s) as overtime. Constraint (24) ensures that staff with morning and evening (L) shift just work at evening shift as overtime in the days which are not assigned to them as L shifts.

Constraint (25) and (26) and (27) ensures supplying of the required number of staff in the morning, evening, and night, respectively. Equation (28) calculates the total working hours of each staff during a month. Constraint (29) ensures compliance with the minimum and maximum limits for working hours for each staff. Equation (30) calculates the total working hours of each staff during off days in a month. Binary variables are stated in equations (31) and (32), and continuous variables are stated in (33).

Table 1 Model Parameters.

α	β	x	f	B	F
0.4	0.35	8	8	11	11

Table 2 Salaries and benefits details per personnel.

Personnel No.	C_i(IRR)	S_i(IRR)
1	111.269	22.458.833
2	107.551	21.875.878
3	106.646	22.029.165
4	107.551	23.617.269
5	105.617	23.114.157
6	103.919	21.500.802
7	103.919	19.826.796
8	103.924	19.610.768
9	101.919	21.167.381
10	97.384	21.907.380
11	88.778	20.379.915
12	88.778	18.963.538
13	84.823	18.353.871
14	84.823	18.079.651
15	84.823	18.354.618
16	84.823	16.896.370

IRR: The currency of Iran

3. Results

In this section, a numerical example for the proposed shift scheduling problem is presented. In this case study, a number of 16 personnel of one section of Tehran Pasargad Hospital with different salaries and benefits are analyzed for a 30 days' period (from December 22, 2019, to January 20, 2020). Thus, the ordered number of days 1 to 30 represent each of mentioned dates. In this period, there are 4 off days in the calendar of the country of study, which are days 6, 13, 20, and 27. The required number of staff in workdays at morning, evening, and night shifts are 7, 4, and 3, respectively. For off days, the required number of staff at morning, evening, and night shifts are 5, 4, and 3, respectively.

Maximum possible overtime working hour of personnel is considered as 100 h. Other information for the model are illustrated in table 1. Moreover, detailed information about salaries and benefits of each 16 staff is showed in table 2. Finally, in Fig. 1 we presented a flowchart to get information for this case study and solve the problem.

The resulting linear programming problem for our proposed scheduling model with this case study is solved with Lingo 18 software on a 64-bit machine with 16 GB memory in 433.88 s and obtained an optimal solution value of 355,951,312 IRR (2542.5 $) for the scheduled period. The schedule for each 16 staff is illustrated in Table 3. This solution reduced the total cost of payments by 10 percent (295.8 $), compared with the manual scheduling conducted by the section manager.

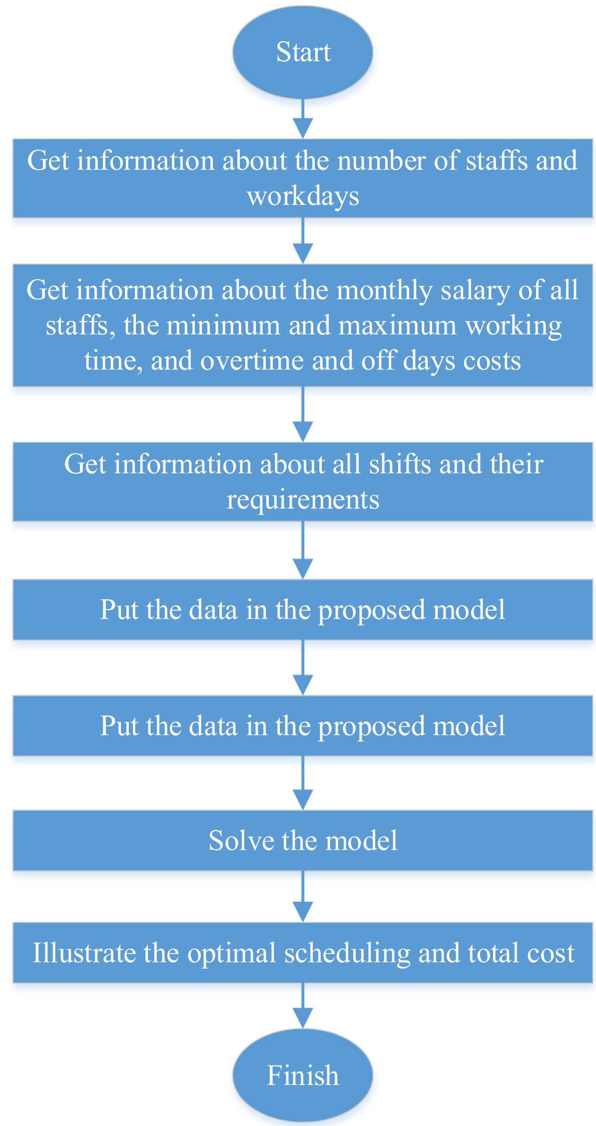

Fig. 1 The flowchart diagram of problem solving method.

4. Sensitivity analysis and discussions

In this section we have used an artificial instance with 6 staffs and 7 days, which there are 1 off day. For all staffs and days, we assumed $S_i = 1200$, $C_i = 20$, $W = 60$, $V = 100$, $E_i = 3$, $O_i = 2$, $P_i = 2$, $F = B = 4$, $x = f = 3$, $\alpha = 0/4$ and $\beta = 0/35$. The resulting artificial problem for the presented model is solved with Lingo 18 software in 1 s and obtained an optimal solution value of 8128 $ for the hypothetical scheduled period. To do this, we changed the values of important parameters, includes total number of staffs, total number of workdays, minimum working time of each staff, maximum working time of each staff, minimum number of staff needed in the morning, minimum number of staff needed in the evening, minimum number of staff needed in the night, percentage added to base salary for working on overtime or holiday, percentage added to base salary for working on over-night and monthly payroll

Table 3 Scheduled shifts for each 16 staff.

Personnel No.	**Shifts**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	**D**	D	D	D	D	D		D	D	D	D	D	D		D	D	D G	D	D G	D		D	D	D G	D	D	D		D	D G	D
2	**D**	D	D	D	D	D		D	D	D	D	D	D G		D G	D	D	D	D	D G		D	D	D	D	D G	D		D	D	D
3	**D**	D	D G	D	D G	D		D	D G	D	D G	D	D		D	D	D	D	D	D		D	D	D	D	D	D		D	D	D
4	**L**		L		L		L		L		L		L		L		L		L		L		L		L		L		L		L
5	**L**		L		L		L		L		L		L		L		L		L		L		L		L		L		L		L
6	**L**	L		L		L		L		L		L		L		L		L		L		L		L		L		L		L	
7	**L**	L		L		L		L		L		L		L		L		L		L		L		L		L		L		L	
8	**L**		L		L		L		L		L		L		L		L		L		L		L		L		L		L		L
9	**L**	L		L		L		L		L		L		L		L		L		L		L		L		L		L		L	
10	**L**	L		L		L		L		L		L		L		L		L			L		L		L		L		L		L
11	**N**		N		N		N		N		N		N		N		N		N		N		N		N		N		N		N
12	**N**	N		N		N		N		N		N		N		N		N		N		N		N		N		N		N	
13	**N**	N	D	N	D	N	D	N	D	N	D	N	D	N	D	N		N		N	D	G N		N		N		N		N	
14	**N**		N		N		G N		N		N		N	D	N		N		N	D	N	D	N		N	D	N	D	N	D	N
15	**N**	N		N		N	D	N		N		N		N		N	D	N	D	N		N		N		N		G N		N	
16	**N**		N		N		N		N		N		N		N		N		N		N		N	D	N		N	D	N		N

Table 4 Sensitivity analysis.

Parameters	% changes	Total Cost (Z)		Total Overtime*	
		Value	% changes	Value	% changes
Initial	0	8128	0	18	0
m	%-50	Infeasible		Infeasible	
	%-25	6688	−17.71	138	666.66
	%25	10,384	27.77	0	−100
	%50	11,584	42.51	0	−100
r	%-50	7872	−3.15	0	−100
	%-25	7760	−4.52	0	−100
	%25	9216	13.33	126	600
	%50	Infeasible		Infeasible	
W	%-50	11,936	46.85	198	1000
	%-25	9391	15.53	103	472.22
	%25	7942	−2.28	18	0
	%50	7721	−5.01	0	−100
E_j	%-50	7984	−1.77	0	−100
	%-25	7984	−1.77	0	−100
	%25	8464	4.13	60	233.33
	%50	8800	8.26	102	466.66
O_j	%-50	7984	−1.77	0	
	%-25	Infeasible		Infeasible	
	%25				
	%50	8464	4.13	60	233.33
P_j	%-50	7592	−6.59	0	−100
	%-25	Infeasible		Infeasible	
	%25				
	%50	9192	13.09	102	466.66
S_i	%-50	4064	−50	18	0
	%-25	6096	−25	18	0
	%25	10,160	25	18	0
	%50	12,192	50	18	0
α	%-50	8056	−0.88	18	0
	%-25	8092	−0.44	18	0
	%25	8164	0.44	18	0
	%50	8200	0.88	18	0
β	%-50	7736	−4.82	18	0
	%-25	7932	−2.41	18	0
	%25	8324	2.41	18	0
	%50	8520	4.82	18	0

* Total overtime $= \sum_{i=1}^{m}(U_i - W)$.

of staff parameters, one by one as we assumed the other parameters to be fixed (see Table 4). As well, the following facts are perceived based on Table 4:

The number of staff (m) has an inverse relationship with the value of total overtimes and is highly sensitive to the changes in the values of total costs because every staff has a minimum working time, and as their number increases, the problem must find the optimal solution which satisfies all the minimum working hours. Despite this condition, it can be concluded this model also helps planners and managers to determine the appropriate number of staff for their scheduling program. Although there are no binary variables for hiring and firing staff, and we just schedule the existing staff in the proposed model, but we can perform a sensitivity analysis on the number of staff, and then the appropriate number is obtained based on its results.

The number of workdays (r) is extremely sensitive to the changes in the values of total costs and overtime. In simple terms, total working hours increases when the number of workdays becomes greater, as a result of that total costs will increase. In such a case, one way to reduce costs can sometimes be to hire new staff to decrease overtime hours. On the other hand, planners may be able to reduce costs by firing some existing staff when the number of working days is decreased. Additionally, if the number of workdays is much more than the number of staffs, the problem can be infeasible because the total working hours of the staffs cannot satisfy all the existing demands.

The minimum working time of each staff (W) highly affects the value of total costs and overtime. For example, if the minimum working time of each staff is reduced, the managers will have to pay more overtime to staff to perform their duties. E_j, O_j, P_j, α, and β are moderately sensitive to the changes in the value of total costs. E_j, O_j, and P_j are highly sensitive to the changes in the value of total overtime. On the other hand, S_i, α, and β are insensitive or often slightly sensitive to the changes in the value of total overtime. But monthly payroll highly affects total costs.

5. Conclusions

In recent years, as the variable costs of the healthcare systems and medical centers are increasing, one of the most important challenges for hospital managers is to optimally schedule staff shifts so that minimum costs are provided while complying with local union laws. There several constraints and limitations in this regard, such as the required number of staff on each shift, available personnel, legislative laws, hospital rules, personnel physical health, etc. Therefore, it is important to have a thorough mathematical model which could be able to formulate all these constraints and considerations.

In this study, a mixed-integer linear programming model is presented for homogeneous personnel shift scheduling. In this mathematical model, staff salaries and benefits, staff number, required number of personnel per shift, hospital and legislative rules are given as the input of the model. Then this model has been applied for a case study problem in a section of Pasargad hospital with 16 personnel. The results indicate a reduction in the costs by 10 percent in comparison with the manual current schedule. Considering this result, we can conclude that if this model is implemented on a larger problem, with the higher number of staff, the resulting cost reduction would be significant. Another important assumption embedded in the model is the physical endurance and satisfaction of the personnel. These factors can be controlled by adding hospital conditions, and the model is flexible and dynamic in this regard.

For further researches, this problem can also be modeled by considering heterogeneous personnel (with different skills), the dynamic requirement of staff in shifts, other costs like dressings and dining, and loyalty of personnel. Besides, the researchers can extend the model without considering the minimum and maximum working time of each staff. On the other hand, they may be able to extend this model by considering binary variables for hiring and firing staff in each period. This proposed model can also be modeled with other objectives like the quality of staff work. Finally, the researchers can adopt their problems with this study and use the rules and conditions of work that apply in their own country and region.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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