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**題組：Problem Set Archive** **with Online Judge**

**題號：10007: Count the Trees**

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**題意：**

有n個node的binary tree，請問有此tree有幾種排列組合方式？

**題意範例：**

Input 2:

Output: 4

Input: 3

Output: 30

**解法：**

將問題分解為二：

(1) 不管node之編號，單純算n個node可有幾種tree之排列

Dynamic Programming

大數加法/乘法

(2) 將node之編號帶入各個tree的排列

排列組合：n!

大數乘法

(3) 將(1)與(2)相乘

若有範例，則列出。若沒有，則填寫「無」。

**解法範例**：

 Input: 2 - (1)

tree有幾種排列：

1個root node

1個other node

Tree之排列：2

Input: 2 - (2)

將編號填入tree中

每一種tree之排列數：2!

Output: 2 \* 2! = 4

Input: 3 - (1)

tree有幾種排列：

1個root node

2個other node

Tree之排列：5

Input: 3 - (2)

將編號填入tree中

每一種tree之排列數：3!

Output: 5 \* 3! = 30

**討論：**

(1) 將計算過之 n! 值儲存，可提升下筆輸入之運算速度

(2) 大數乘法時，需注意可能是兩個大數之相乘

**程式：**

#include <iostream>

#include <cstring>

#include <cstdlib>

#include <vector>

#include <fstream>

//#define TEST

using namespace std;

typedef struct NODE{

 vector<char> tree;

 vector<char> fact;

}NODE;

void init();

void print(vector<char> str);

void tree\_count(short int n);

void fact\_count(short int n);

void add(vector<char>& d, vector<char> s);

vector<char> multiply(vector<char> a, vector<char> b);

NODE record[301];

ofstream file;

int main(void)

{

 short int choice = 0;

 vector<char> result;

 #ifdef TEST

 file.open("file.txt", ios::out | ios::trunc ); //why cannot put in global

 #endif

 init();

 do{

 cin >> choice;

 if(!choice) continue;

 tree\_count(choice);

 //cout << "tree:"; print(record[choice].tree); cout << "---------\n";

 fact\_count(choice);

 //cout << "fact:"; print(record[choice].fact); cout << "---------\n";

 result = multiply(record[choice].tree, record[choice].fact);

 print(result);

 }while(choice);

 return 0;

}

void init()

{

 record[0].tree.push\_back('1');

 record[0].fact.push\_back('1');

 return;

}

void print(vector<char> str)

{

 for(int i = str.size()-1; i >= 0; i--){

 #ifndef TEST

 cout << str[i];

 #endif

 #ifdef TEST

 file << str[i];

 #endif

 }

 #ifndef TEST

 cout << endl;

 #endif

 #ifdef TEST

 file << endl;

 #endif

 return;

}

void tree\_count(short int n)

{

 if(!record[n].tree.empty()) return;

 vector<char> temp;

 for(short int i = 0; i < n>>1; i++){ //i.e., i < (n-1)/2 +(n-1)%2

 tree\_count(i);

 tree\_count(n-1-i);

 add(record[n].tree, multiply(record[i].tree, record[n-1-i].tree) ); //may have problem

 //cout << \_\_LINE\_\_ << ":"; print(record[n].tree); //

 }

 vector<char> multiplicand(1, '2');

 record[n].tree = multiply(record[n].tree, multiplicand);

 //cout << \_\_LINE\_\_ << ":"; print(record[n].tree); //

 if(n%2){

 tree\_count(n>>1);

 temp = multiply(record[n>>1].tree, record[n>>1].tree);

 //cout << \_\_LINE\_\_ << ":"; print(temp); //

 add(record[n].tree, temp);

 //cout << \_\_LINE\_\_ << ":"; print(record[n].tree); //

 temp.clear();

 //add(record[n].tree, multiply(record[n>>1].tree, record[n>>1].tree) ); //may have problem

 }

 return;

}

void fact\_count(short int n)

{

 if(!record[n].fact.empty()) return;

 fact\_count(n-1);

 vector<char> multiplicand;

 short int t = n;

 while(t){

 multiplicand.push\_back('0' + (t%10));

 t /= 10;

 }

 record[n].fact = multiply(record[n-1].fact, multiplicand);

 return;

}

void add(vector<char>& d, vector<char> s) //add s to d, do not allocate

{

 //cout << "d.capacity: " << d.capacity() << endl << "s.capacity: " << s.capacity() << endl;

 if(d.size() < s.size()){

 for(int i = d.size(); i < s.size(); i++) d.push\_back('0');

 }

 else if(d.size() > s.size()){

 for(int i = s.size(); i < d.size(); i++) s.push\_back('0');

 }

 //cout << "d.capacity: " << d.capacity() << endl << "s.capacity: " << s.capacity() << endl;

 char carry = 0, temp = 0;

 for(short int i = 0; i < d.size(); i++){

 temp = d[i] + s[i] - 2\*'0' + carry;

 carry = temp/10;

 d[i] = '0' + (temp%10);

 }

 if(carry) d.push\_back('0' + carry);

 //cout << "d.capacity: " << d.capacity() << endl << "s.capacity: " << s.capacity() << endl;

 return;

}

vector<char> multiply(vector<char> a, vector<char> b) //returning muliple of a and b, allocate

{

 vector<short int> temp; //may need int

 vector<char> result;

 temp.assign(a.size() + b.size(), 0);

 result.assign(a.size() + b.size() - 1, '0');

 for(int i = 0; i < a.size(); i++){

 for(int j = 0; j < b.size(); j++){

 temp[i+j] += (a[i]-'0') \* (b[j]-'0'); //add comparison with 0 to speedup?

 }

 }

 short int carry = 0;

 for(int i = 0; i < temp.size() - 1; i++){

 temp[i] += carry;

 result[i] = '0' + (temp[i]%10);

 carry = temp[i]/10;

 }

 if(carry) result.push\_back('0' + carry);

 //cout << "result.capacity: " << result.capacity() << endl;

 temp.clear();

 return result;

}