Scheduling Packet Transmissions on Logical Topologies in WDM Optical Star Networks

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Abstract
In this paper, we address the design of multi-hop packet transmissions for the All-to-All Broadcast (AAB) problem in a Wavelength Division Multiplexing (WDM) optical star network with \(N\) nodes. The tunable transmitter on each node needs a non-negligible tuning latency \(\delta\) to switch between wavelengths. To decrease the affection of the packet delay in multi-hop scheduling, the maximum of hop distance of each packet transmission is limited to a constant \(\rho\). We propose an improved multi-hop scheduling algorithm to shorten the duration of scheduling periods. The number of tuning operations on each node is at most \(\left\lceil\frac{2N}{\rho+1}\right\rceil-1\) and the schedule length of our multi-hop scheduling is at most \((\left\lceil\frac{2N}{\rho}\right\rceil-1)(\frac{\rho (\rho-1) }{2} + \delta)\) if \(\rho\) is odd. Otherwise, the tuning time on each node is at most \(\left\lceil\frac{2N}{\rho}\right\rceil-1\) and the schedule length is at most \((\left\lceil\frac{2N}{\rho}\right\rceil-1)(\frac{\rho (\rho-1) }{2} + \delta)\). For general case, when \(\rho < \frac{1 + \sqrt{1 + 8\delta + 6\delta^2}}{2}\), the schedule length of our multi-hop scheduling algorithm is shorter than that of the optimal single-hop scheduling algorithm. This also improves the previous results on the multi-hop scheduling length of the AAB problem.

1 Introduction
Packet scheduling provides a way to reduce routing control and guarantee overall throughput. An interesting special case of the packet scheduling is all-to-all broadcast (AAB) in which every transmitter/receiver pair has exactly one packet to be transferred. In this paper, we consider the AAB problem in Wavelength Division Multiplexing (WDM) optical star networks with \(N\) nodes and \(w\) wavelengths. If each node is equipped with one tunable transmitter and one fixed receiver (TT/FR), the connectivity between nodes can be achieved by tuning transmitters to the different receiving wavelengths [1]. However, the tuning time cannot be neglected with respect to the packet transmission time. With some of the optical components available today, the tuning latencies of tunable lasers need a few ms. It is much longer than the packet transmission time which is almost less than a fraction of \(\mu s\) [7].

In [4], Yeh et al. proposed an optimal single-hop scheduling algorithm with schedule length \(\max\{ (N-1)\left\lceil\frac{N}{w}\right\rceil, N-1 + w\delta \}\) for the AAB problem, where \(\delta\) is the tuning latency of the tunable transmitter. The schedule length of the optimal single-hop scheduling algorithm is seriously affected by the non-negligible tuning latency because each node must do \(w\) times of tuning actions in an all-to-all broadcast. For reducing the number of tuning operations and shorten the duration of schedule length, the multi-hop transmission scheduling is another method for solving the AAB problem. Each packet transmission can be sent though some intermediated nodes from source to destination to reduce the number of tuning operations on each node [2].

For example, we consider the AAB problem in the WDM optical star network with 7 nodes as Figure 1(a) in which the receiving wavelength of node \(i\) is fixed as \(w_i\), \(0 \leq i \leq 6\). By using the single-hop scheduling algorithm in [4], the
optimal schedule length is $7\delta+6$ time slots. If the maximum hop distance is restricted to 3 and each node $i$ tunes its transmitting wavelength to $w_{(i+1 \mod 7)}$ for constructing a logical topology as Figure 1(b). We schedule all packet transmissions with one, two and three hop distance separately in Table 1. Next, each node $i$ tunes its transmitting wavelength to $w_{(i-1 \mod 7)}$ for constructing another topology in Figure 1(c). Packet transmissions can be scheduled similarly as Table 1. The schedule length of multi-hop scheduling is $2\delta+12$ which is shorter than the length of optimal single-hop scheduling when $\delta > \frac{6}{\gamma}$. Obviously, the schedule length can be reduced by using the multi-hop scheduling.

We have proposed a multi-hop scheduling algorithm for the AAB problem in [9]. We constructed logical ring topologies for simplifying the multi-hop routing, and restricted the maximum hop distance of packet transmissions to minimize the packet delay. In the paper, we will improve the previous results [9] by designing better logical topologies to minimize the number of tuning actions on transmitters. The problem of Logical Topology Construction with maximum $\rho$ hop distance is called the $\rho$-LTC problem. The objective of the $\rho$-LTC problem is to minimize the number of logical topologies for reducing the number of tuning actions. In [9], we partitioned $N$ nodes into groups with size either $\rho$ or $\rho+1$, and designed $2\lceil\frac{N-1}{\rho}\rceil$ logical rings at most for the $\rho$-LTC problem. However, the result is not close to the optimal solution ($\lceil\frac{N-1}{\rho}\rceil$) of the $\rho$-LTC problem.

In this paper, the main improvement is that we partition $N$ nodes into $x$ groups and the size of groups is reduced to either $\frac{\rho}{2}$ or $\frac{\rho+1}{2}$. We can simplify the design of logical topologies by considering only these $x$ groups in Section 3. If $x$ is odd, we transfer the problem to the Hamiltonian cycles problem on a complete directed graph with $x$ vertices, and propose an algorithm to obtain $x-1$ logical rings. The number of logical topologies by our method is at most $\lceil\frac{2N}{\rho+1}\rceil - 1$. On the other hand, if $x$ is even, we transfer the problem to the round robin tournament problem with $x$ players, and apply the tournament algorithm for constructing $x-1$ logical topologies. The number of logical topologies by our method is at most $\lceil\frac{2N}{\rho}\rceil - 1$.

In [5], Marsan et al. proposed a multi-hop scheduling algorithm for the general scheduling problem. Especially, when the tuning latency is large enough and the hop distance of packet transmissions is at most 3, their schedule length is shorter than that of single-hop scheduling algorithm. In Section 5, as $\rho = 3$, we will produce $\lceil\frac{N}{3}\rceil - 1$ logical topologies that is 1.5 times the optimal solution of the $\rho$-LTC problem. In fact, our multi-hop scheduling needs $\lceil\frac{N}{2}\rceil - 1$ tuning actions on each node for the 3-LTC problem, and its schedule length is $\lceil\frac{N}{2}\rceil - 1(6 + \delta)$. When $\delta$ is larger than or equal to 4 slots ($\delta \geq 4$), the schedule length is smaller than the optimal schedule length $N\delta + 1 - N - 1$ of single-hop scheduling. In general case, when $\rho < \frac{1+\delta+\sqrt{4\delta^2+16\delta+1}}{2}$, the multi-hop schedule length ($\lceil\frac{N}{\rho+1}\rceil - 1(\frac{\rho(\rho+1)}{2} + \delta)$ will be less than the optimal schedule length $\max\{(N-1)\lceil\frac{N}{w}\rceil, N - 1 + w\delta\}$ of single-hop scheduling in [4]. The result also improves previous results in [9].

2 Problem Definition

We consider the AAB problem in WDM optical star networks with $N$ nodes and $w$ wavelengths, which each node must transmit one packet to other nodes. So there are $N(N - 1)$ packet transmissions to have to be sent in WDM networks. To shorten the schedule length for the AAB problem, we adopt the multi-hop schedule protocol for solving this problem. Moreover, to avoid the influence of transmission delay, we restrict the maximum hop distance of packet transmission to a constant $\rho$. For simplifying the design of logical topologies, we just consider the case when $N = w$. For the general case, $N > w$, our scheduling can be easily extended for it. In slotted schedule systems, packets have fixed length so that a packet transmission time equals one time slot, and we assume that the tuning latency for each transmitter to tune from one wavelength to another needs $\delta$ time slots.

The $\rho$-AAB problem is described as follows: In WDM optical star networks with $N$ nodes and $N$ wavelengths, we address the AAB problem by the multi-hop scheduling model, and the hop distance of each packet transmission is $\rho$ at most. Our objective is to minimize the schedule length.

2.1 Our Multi-hop Scheduling Algorithm

Our multi-hop scheduling algorithm is generally divided into two stages: (1) design logical topology, and (2) schedule packet transmissions [2]. In the first part, to simplify the routing method and to decrease the affection of tuning delay, we assume that all transmitters do tuning operations synchronously in a multi-hop scheduling period. After a tuning operation of each transmitter, we
can construct a logical topology by transmitting wavelengths of all nodes. For example, we construct four logical topologies in Figure 2 for the 3-AAB problem. The logical topology $r_1$ in Figure 2(a) is created by that each node $i$ tunes the transmitting wavelength to $w(i+1 \mod 11)$ simultaneously. Similarly, logical topologies $r_2$, $r_3$ and $r_4$ in Figure 2 are constructed by that each node $i$ separately tunes the transmitting wavelength to $w(i+2 \mod 11)$, $w(i-1 \mod 11)$ and $w(i-2 \mod 11)$.

In the second step, we adopt the pipeline transmission mechanism [3] on each logical topology. The transmission period on each logical topology is divided into $\rho$ stages. On the logical topology $r_2$, we schedule the pipeline transmission with one hop distance in Figure 2(b), two hop distance in Figure 3 and three hop distance in Figure 4. Therefore, the schedule of pipeline transmissions on $r_2$ is shown in Table 2. Obviously, packet transmissions with $k$ hops are sent in the $k$-th stage, and it will takes $\frac{\rho(\rho+1)}{2}$ time slots to process all packets transmissions on each logical topology. If there are $\gamma$ logical topologies for the $\rho$-AAB problem, the schedule length is equal to $\gamma(\frac{\rho(\rho+1)}{2} + \delta)$.
Table 2: Packet scheduling for the 3-AAB problem on the logical topology $r_2$ as Figure 2(b)

<table>
<thead>
<tr>
<th>nodes</th>
<th>Tuning</th>
<th>On Figure 2 (b)</th>
<th>Tuning</th>
</tr>
</thead>
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<tr>
<td></td>
<td>slot $\delta$</td>
<td>1-st stage(1 hop)</td>
<td>2-nd stage(2 hop)</td>
</tr>
<tr>
<td>0</td>
<td>$\delta + 1$</td>
<td>0 $\rightarrow$ 2</td>
<td>0 $\rightarrow$ 2</td>
</tr>
<tr>
<td>1</td>
<td>$\delta + 2$</td>
<td>1 $\rightarrow$ 3</td>
<td>1 $\rightarrow$ 3</td>
</tr>
<tr>
<td>2</td>
<td>$\delta + 3$</td>
<td>2 $\rightarrow$ 4</td>
<td>2 $\rightarrow$ 4</td>
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<td>3</td>
<td>$\delta + 4$</td>
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<td>4 $\rightarrow$ 6</td>
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<tr>
<td>5</td>
<td>$\delta + 6$</td>
<td>5 $\rightarrow$ 7</td>
<td>5 $\rightarrow$ 7</td>
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<tr>
<td>6</td>
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<tr>
<td>7</td>
<td>$\delta + 8$</td>
<td>7 $\rightarrow$ 9</td>
<td>7 $\rightarrow$ 9</td>
</tr>
<tr>
<td>8</td>
<td>$\delta + 9$</td>
<td>8 $\rightarrow$ 10</td>
<td>8 $\rightarrow$ 10</td>
</tr>
<tr>
<td>9</td>
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<tr>
<td>10</td>
<td>$\delta + 11$</td>
<td>10 $\rightarrow$ 12</td>
<td>10 $\rightarrow$ 12</td>
</tr>
</tbody>
</table>

3 Logical Topology Design

As $\delta$ is large enough, the schedule length of the multi-hop transmission scheduling is decided by the logical topology design. This section focuses on the design of logical topologies, and the logical topologies construction problem with maximum $\rho$ hop distance is called $\rho$-LTC problem.

The $\rho$-LTC problem is described as follows: In WDM optical star networks with $N$ nodes and $N$ wavelengths, construct a set of logical topologies $LT = \{l_1, l_2, \ldots, l_m\}$ for each pair of nodes $(i, j)$ such that $\text{Dist}(i, j) \leq \rho$. Note $\text{Dist}(i, j)$ is the minimal number of hop distance from node $i$ to node $j$ in logical topologies $LT$. The objective is to minimize the number $m$ of logical topologies. 

The lower bound of the $\rho$-LTC problem is at least $\lceil \frac{2^N}{\rho^2} \rceil$ logical topologies [9].

For constructing logical topologies, we partition $N$ nodes into $x$ groups, $g_0, g_1, \ldots, g_{x-1}$. If the maximum hop distance $\rho$ is even, let $x = \lceil \frac{N}{\rho^2} \rceil = \lceil \frac{2^N}{\rho^2} \rceil$. Nodes are divided into $\lceil \frac{2^N}{\rho^2} \rceil$
groups and \( G^\rho = \{ g_0, g_1, ..., g_{\lceil \frac{2N}{\rho} \rceil - 1} \} \) where \( g_i = (i\rho, i\rho + 1, i\rho + 2, ..., (i + 1)\rho - 1) \), for all \( i \in \{0, 1, ..., \lceil 2N/\rho \rceil - 2\} \), and \( g_{\lceil \frac{2N}{\rho} \rceil - 1} = ((\lceil \frac{2N}{\rho} \rceil - 1)\rho/2, ((\lceil \frac{2N}{\rho} \rceil - 1)\rho/2 + 1, ..., N - 1) \). When \( \rho = 4 \) and \( N = 9 \), \( x = \lceil \frac{2N}{\rho} \rceil = 5 \). Then \( g_0 = \{0, 1\} \), \( g_1 = \{2, 3\} \), \( g_2 = \{4, 5\} \), \( g_3 = \{6, 7\} \) and \( g_4 = \{8\} \).

If \( \rho \) is odd, let \( x = \lceil \frac{N}{(\rho + 1)/2} \rceil = \lceil \frac{N}{\rho} \rceil \). We divide \( N \) nodes into \( \lceil \frac{2N}{\rho + 1} \rceil \) groups and \( G^\rho = \{ g_0, g_1, ..., g_{\lceil \frac{2N}{\rho + 1} \rceil - 1} \} \) where \( g_i = (i\rho + 1/2, i\rho + 1 + 1, i\rho + 1 + 2, ..., (i + 1)\rho + 1 - 1) \), for all \( i \in \{0, 1, ..., \lceil 2N/(\rho + 1) \rceil - 2\} \), and \( g_{\lceil \frac{2N}{\rho + 1} \rceil - 1} = ((\lceil \frac{2N}{\rho + 1} \rceil - 1)\rho + 1/2, (\lceil \frac{2N}{\rho + 1} \rceil - 1)\rho + 2 + 1, ..., N - 1) \). For the case, \( \rho = 5 \) and \( N = 17 \), \( x = \lceil \frac{2N}{\rho + 1} \rceil = 6 \). So \( g_0 = \{0, 1, 2\} \), \( g_1 = \{3, 4, 5\} \), \( g_2 = \{6, 7, 8\} \), \( g_3 = \{9, 10, 11\} \), \( g_4 = \{12, 13, 14\} \) and \( g_5 = \{15, 16\} \).

Assume that \( g_i = (a_1, a_2, ..., a_{\rho}) \) and \( g_j = (b_1, b_2, ..., b_{\rho}) \). When there exists a connection from \( g_i \) to \( g_j \) on a logical topology, the order of nodes is \( a_1, a_2, ..., a_{\rho}, b_1, b_2, ..., b_{\rho} \) on it.

**Lemma 3.1.** If nodes in any two groups \( g_i \) and \( g_j \) are connected with a logical topology \( L \), all nodes in \( g_i \) can send packets through \( L \) hop distance at most to all nodes in \( g_j \) on \( L \), where \( i, j \in \{0, 1, ..., x - 1\} \).

**Proof.** If \( \rho \) is odd, each group has at most \((\rho + 1)/2\) nodes. Otherwise, each group has at most \(\rho/2\) nodes. Even in an extreme case when the logical topology \( L \) is linear, the maximum hop distance between any two nodes is at most \( \rho \).

For applying the condition of Lemma 3.1 to the \( p-LTC \) problem, we need to ensure that each group in \( G^\rho \) is adjacent to all other groups on some logical topologies. If the number of groups in \( G^\rho \) is even, we reduce the \( p-LTC \) problem to Hamiltonian Cycles (HC) problem. Otherwise, we reduce the \( p-LTC \) problem to Round Robin Tournament (RRT) problem when \( x \) is odd. Our results show that \( x - 1 \) logical topologies is sufficient to solve the \( p-LTC \) problem, i.e. we need \( \lceil 2N/\rho \rceil - 1 \) logical topologies when \( \rho \) is even and \( \lceil 2N/(\rho + 1) \rceil - 1 \) logical topologies when \( \rho \) is odd.

### 3.1 Design by Hamiltonian Cycles (HC)

When the number \( x \) of groups is odd, we reduce the \( p-LTC \) problem to the HC problem and design logical rings to satisfy the condition of Lemma 3.1. We transfer \( G^\rho \) to a complete directed graph \( D \) as follows. Each group \( g_i \) in \( G^\rho \) is represented as a vertex in \( D \) and \( g_i \) is adjacent to \( g_j \) for \( 0 \leq i, j \leq x - 1 \) and \( i \neq j \). In Figure 5, there is a complete directed graph \( D \) with five groups for \( \rho = 4 \) and \( N = 9 \).

In [6], Chartrand and Lesniak proposed several methods to create all Hamiltonian cycles on a complete undirected graph. They also indicated that if \( x \) is odd, there are \( \frac{x - 1}{2} \) Hamiltonian cycles on a complete undirected graph. Since the logical topology in the multi-hop transmission should be directed, by applying methods in [6], we propose the Logical Topologies Construction (LTC) Algorithm to construct \( x - 1 \) logical rings for the \( p-LTC \) problem as follows.

**LTC-Odd Algorithm**

1. Ignore the direction of edges on \( D \) and apply methods in [6] to create \( \frac{x - 1}{2} \) Hamiltonian cycles for \( x \) vertices. Assume that we obtain the set \( \{H_i|1 \leq i \leq \frac{x - 1}{2}\} \) of all Hamiltonian cycles.
2. Add the direction to all edges of all Hamiltonian cycles in \( \{H_i|1 \leq i \leq \frac{x - 1}{2}\} \). We have the logical ring set \( \{r_i|1 \leq i \leq \frac{x - 1}{2}\} \), where we denote the logical ring \( r_i \) as \( \ll g_{i0}, g_{i1}, g_{i2}, g_{i3}, \ldots, g_{i(x-2)}, g_{i(x-1)} \gg \).
3. Reverse the direction of edges on each logical ring

\[ r_i = \ll g_{i0}, g_{i1}, g_{i2}, g_{i3}, \ldots, g_{i(x-2)}, g_{i(x-1)} \gg \] in \( \{r_i\} \), then we obtain another logical ring \( \ll g_{i(x-1)}, g_{i(x-2)}, g_{i(x-3)}, g_{i(x-4)}, \ldots, g_{i1}, g_{i0} \gg \) for \( 1 \leq i \leq \frac{x - 1}{2} \).

In Figure 5, we find two Hamiltonian cycles, \( H_1 = (g_0, g_1, g_2, g_3, g_4) \) and \( H_2 =\)
(g₀, g₂, g₄, g₁, g₃). We design r₁ = ⪯ g₀, g₁, g₂, g₃ ≫ = {0, 1, 2, 3, 4, 5, 6, 7, 8} by H₁, and construct r₂ = ⪯ g₀, g₂, g₄, g₁, g₃ ≫ = {0, 1, 4, 5, 8, 2, 3, 6, 7} by H₂. r₁ is designed as ⪯ 8, 7, 6, 5, 4, 3, 2, 1, 0 ≫ by r₁, r₂ is constructed as ⪯ 8, 7, 6, 3, 2, 8, 5, 4, 1, 0 ≫ by r₂.

When x is odd, the LTC-Odd Algorithm creates x − 1 logical topologies with two logical ring sets {rᵢ|1 ≤ i ≤ ⪯ x−1} and {rᵢ|1 ≤ i ≤ ⪯ x−1} for the ρ-LTC problem. On each logical ring rᵢ, 1 ≤ i ≤ (x−1)/2, nodes in group gᵢ are clockwise connected with nodes in gᵢ+1 for 1 ≤ i ≤ x−1. On the other hand, nodes in groups gᵢ and gᵢ−1 are also clockwise connected on each logical ring rᵢ, 1 ≤ i ≤ (x−1)/2.

3.2 Design by Round Robin Tournament (RRT)

When the number x of groups is even, we reduce the ρ-LTC problem to the RRT problem with x players to produce logical topologies. In the RRT problem, we need to schedule a tournament such that each player plays once with every other person in finite rounds. If x is even, there are x − 1 rounds for the RRT problem with x players [8]. Let Rᵢ be a round \{gᵢ₀, gᵢ₁, gᵢ₂, ..., gᵢᵢ−₁\} of the RRT problem with x players, where i₀ < i₂ < ... < iᵢ−₂, and \{(i₀, i₁, ..., iᵢ−₁)\} is one permutation of \(0, 1, ..., x−1\). For the case, \(ρ = 5\) and \(N = 17\), we solve the RRT problem with six players and schedule the tournament in Table 3. There are five rounds and each round has three playing games. The round 3 is represented as R₃ = \{(g₀, g₁, g₂, g₃, g₄, g₅)\}.

We translate these x − 1 rounds of the RRT problem to logical topologies for the ρ-LTC problem as follows. On each round, each player pair \(gᵢᵢ₀, gᵢᵢ₁\) represents a small logical ring \(≪ gᵢᵢ₀, gᵢᵢ₁ ≫\) in which nodes in group gᵢᵢ₀ are clockwise connected with nodes in gᵢᵢ₁. Let Lᵢ be the logical topology constructed by Rᵢ. Since there are x/2 player pairs in each round, each logical topology should contain with x/2 small logical rings. Let Lᵢ = \{≪ gᵢᵢ₀, gᵢᵢ₁ ≫, ≪ gᵢᵢ₁, gᵢᵢ₂ ≫, ..., ≪ gᵢᵢᵢ₋₂, gᵢᵢ−₁ ≫\}. For example, the logical topology of round R₃ in Table 3 is represented as L₃ = \{≪ g₀, g₂ ≫, ≪ g₁, g₃ ≫, ≪ g₃, g₄ ≫, ≪ g₀, g₁ ≫, ≪ g₂, g₃ ≫, ≪ g₁, g₂ ≫\}. In Figure 6, there are three small rings on the logical topology L₃. Next we propose the LTC-Even Algorithm to construct x − 1 logical rings for the ρ-LTC problem as follows.

**LTC-Even Algorithm**

1. Establish \((x−1)\) rounds in the RRT problem with x player, and let \{Rᵢ|1 ≤ i ≤ \((x−1)\)\} be the set of finite rounds.

2. For each round \(Rᵢ, 1 ≤ i ≤ x−1\), construct a logical topology \(Lᵢ\) with \(x/2\) small logical rings. Let \{Lᵢ|1 ≤ i ≤ \((x−1)\)\} be the set of \(x−1\) logical topologies and \(Lᵢ = \{≪ gᵢ₀, gᵢ₁ ≫, ≪ gᵢ₁, gᵢ₂ ≫, ..., ≪ gᵢᵢ₋₂, gᵢᵢ−₁ ≫\}\).

4 Packet Scheduling

In this section, we schedule packet transmissions of the AAB problem actually on logical topologies, and compute the schedule length of our multi-hop scheduling algorithm.

4.1 Packet Scheduling on Logical Rings

When the number x of groups is odd, the LTC-Odd Algorithm has designed \((x−1)\) logical topologies with \{rᵢ|1 ≤ i ≤ \((x−1)\)\} and \{rᵢ|1 ≤ i ≤ \((x−1)\)\} for the ρ-LTC problem. Assume that nodes in each logical ring \(rᵢ\) are clockwise connected as ≪ \(nᵢ₀, nᵢ₁, ..., nᵢᵢ−₁ ≫\). Similarly, nodes in \(rᵢ\) are also clockwise connected as ≪ \(nᵢᵢ₋₂, nᵢᵢ₋₁, ..., nᵢ₀ ≫\), where \(1 ≤ i ≤ \((x−1)/2\)\). We apply the pipeline transmission scheduling [3] to transmit packets on logical rings as follows.

**Scheduling Algorithm on Logical Rings**

1. On each logical ring \(rᵢ\) in \{rᵢ|1 ≤ i ≤ \((x−1)/2\)\}, each node \(nᵢᵢ \) transmits its transmitting wavelength to the receiving wavelength of \(nᵢᵢ₊₁ \mod N\) for \(0 ≤ j ≤ N−1\). Now each node \(nᵢᵢ \) transmits a packet through k hop distance to \(nᵢᵢ₊k \mod N\) for \(0 ≤ j ≤ N−1\) and \(1 ≤ k ≤ g\), where \(g = ρ \) if \(ρ\) is odd and \(g = ρ − 1\) otherwise.

2. On each logical ring \(rᵢ\) in \{rᵢ|1 ≤ i ≤ \((x−1)/2\)\}, each node \(nᵢᵢ \) transmits its transmitting wavelength to the receiving wavelength of \(nᵢᵢ−1 \mod N\) for \(0 ≤ j ≤ N−1\). Similarly, each node \(nᵢᵢ \) transmits a packet through k hop distance to \(nᵢᵢ−k \mod N\) for \(0 ≤ j ≤ N−1\) and \(1 ≤ k ≤ g\), where \(g = ρ \) if
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On each logical ring $r_i$, the scheduling period is partitioned into $g$ stages, where $g \leq \rho$. Each packet transmission with $k$ hop distance is sent in the $k$-th stage, where $1 \leq k \leq g$. It takes at most $\frac{\rho(x+1)}{2}$ time slots for sending packet transmissions on $r_i$. Since the tuning time is $\delta$, the schedule length on $\{r_i\}$ is at most $(x-1)(\frac{\rho(x+1)}{2} + \delta)$. At the same reason, the schedule length on $\{t_i\}$ is also $(x-1)(\frac{\rho(x+1)}{2} + \delta)$ at most. Thus, the total is $(x-1)(\frac{\rho(x+1)}{2} + \delta)$ at most when $x$ is odd.

4.2 Packet Scheduling on Logical Group Pairs

When the number $x$ of groups is even, the LTC-Even algorithm already constructs a set $\{L_i\}$ of logical topologies, where $L_i = \{\langle g_{i0}, g_{i1} \rangle, \langle g_{i2}, g_{i3} \rangle, \ldots, \langle g_{i(x-2)}, g_{i(x-1)} \rangle\}$. On each logical topology, packet transmissions are scheduled on an individual group pair (small logical ring) independently. We assume that nodes in a group pair $\langle g_{ij}, g_{i(j+1)} \rangle$ are clockwise connected as $\langle n_{ij}^{t_1}, n_{ij}^{t_2}, \ldots, n_{ij}^{t_y} \rangle$ to form a small logical ring, where $i$ is even then $\rho = \rho - 1$ else $\rho = \rho$. Now, by applying the pipeline transmission scheduling, we describe the scheduling algorithm on these logical topologies as follows.

<table>
<thead>
<tr>
<th>Round 1($R_1$)</th>
<th>Round 2($R_2$)</th>
<th>Round 3($R_3$)</th>
<th>Round 4($R_4$)</th>
<th>Round 5($R_5$)</th>
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<td>$(g_4, g_5)$</td>
<td>$(g_1, g_4)$</td>
<td>$(g_1, g_3)$</td>
<td>$(g_1, g_2)$</td>
</tr>
</tbody>
</table>

Figure 6: Three small rings on the logical topology $L_3$

$\rho$ is odd and $g = \rho - 1$ otherwise.

Scheduling Algorithm on Logical Group Pairs

1. On each logical topology $L_i$ in $\{L_i\}$, each node $n_i^{t_j}$ tunes its transmitting wavelength to the receiving wavelength of $n_i^{t_{j+1} \mod \rho}$ for $0 \leq t \leq g - 1$ and $j \in \{0, 2, \ldots, x-2\}$.

2. On each small logical ring $\langle n_0^{t_j}, n_1^{t_j}, \ldots, n_6^{t_j} \rangle$ in $L_i$, each node $n_i^{t_j}$ transmits a packet through $k$ hop distance to $n_i^{t_{j+k \mod \rho+1}}$ for $0 \leq t, k \leq g$ and $j \in \{0, 2, \ldots, x-2\}$.

On $\langle n_0^{t_j}, n_1^{t_j}, \ldots, n_6^{t_j} \rangle$, each node transmits $k$ packets through hop distance $k$ to destinations separately. The transmission period on the scheduling is divided into $k$ stages and each packet transmission with $k$ hop distance is sent in the $k$-th stage. Since $g \leq \rho$, it takes at most $\frac{\rho(x+1)}{2}$ time slots for sending packets. Although there are $\frac{x}{2}$ small logical rings on each logical topology $L_i$ in $\{L_i\}$, the pipeline transmission on each ring is processed independently. So, the total schedule length is at most $(x-1)(\frac{\rho(x+1)}{2} + \delta)$ where $x$ is even.
5 Performance

No matter which scheduling algorithm we use, the maximum schedule length is always \( (x - 1)(\frac{2N}{p} + \frac{1}{2} + \delta) \) time slots, where \( x = \lceil \frac{2N}{p} + 1 \rceil \) and \( \delta = p \) if \( p \) is odd; otherwise, \( x = \lceil \frac{2N}{p} \rceil \) and \( \delta = p - 1 \). Clearly, the number of logical topologies is \( \lceil \frac{2N}{p} \rceil - 1 \) as \( p \) is odd. And, the number of logical topologies is \( \lceil \frac{2N}{p} \rceil \) if \( p \) is even. Thus, the schedule length of our multi-hop scheduling is as follows:

- The schedule length is \( \lceil \frac{2N}{p} \rceil - 1 \) as \( p \) is odd.
- The schedule length is \( \lceil \frac{2N}{p} \rceil \) as \( p \) is even.

The schedule length is shorter than the schedule length \( 2(\lceil \frac{N}{p} \rceil)(\frac{p(\rho+1)}{2} + \delta) \) in [9]. Moreover, when \( p = 3 \), the number \( 2(\lceil \frac{N}{p} \rceil) \) of logical topologies by [9] is almost close to \( N - 1 \). Our method constructs only \( \lceil \frac{N}{2} \rceil - 1 \) logical topologies which can approach to 1.5 times the lower bound \( \lceil \frac{N-1}{2} \rceil \).

When the tuning latency on each transmitter is greater than or equal to 4 time slots, the multi-hop schedule length \( (\lceil N/2 \rceil - 1)(6 + \delta) \) should be shorter than that of optimal single-hop scheduling in [4]. For general case, when \( p < \frac{1 + \delta + \sqrt{\delta^2 - 4\delta + 4}}{2} \), our multi-hop scheduling algorithm is better than the optimal single-hop scheduling algorithm.

We get done the \( \rho \)-LTC scheduling algorithms, the single-hop scheduling algorithm, and Logical Ring Construction algorithms (\( \rho \)-LRC) [9]. Then we use numerations to verify the schedule length with these scheduling algorithms, where \( \rho \) is from 2 to 31. The network is considered with 32 nodes and 32 available wavelengths. The performances are plotted in Figure 7, Figure 8 and Figure 9 as curves of the schedule length versus the number \( \delta \) of slots to tune transmitters which is restricted to between 20 to 100.

In Figure 7, we first compare the \( \rho \)-LTC schedule length with the optimal single-hop schedule length. The performance curves in the figure show that the schedule length of \( \rho \)-LTC multi-hop scheduling algorithms is almost shorter than \( N - 1 + N\delta \) of the optimal single-hop scheduling algorithms in [4]. That is to say, the multi-hop scheduling is better than the single-hop scheduling when the tuning latency cannot be ignored in scheduling. However, there exists a case \( \rho = 2 \) which schedule length is close to the single-hop schedule length because the time of tuning operations in scheduling is almost equal to \( N \).

In Figure 8 and Figure 9, we compare the schedule length by the \( \rho \)-LTC scheduling algorithms with the previous \( \rho \)-LRC schedule length in [9]. When the hop distance is limited to even values as 2,4, the curves in Figure 8 mention that \( \rho \)-LTC schedule lengths are almost equal to \( \rho \)-LRC lengths in [9], but they are still smaller than \( \rho \)-LRC lengths. We also consider the odd values as 3,5 of hop distance for the AAB problem in Figure 9. The curves in Figure 9 indicate that each \( \rho \)-LTC schedule length is properly less than that of \( \rho \)-LRC algorithms in [9].

Although we just use eight values of hop distance for our multi-hop scheduling algorithms in Figure 7, Figure 8 and Figure 9, \( \rho \) is considered from 2 to 31 really. Our multi-hop \( \rho \)-LTC scheduling algorithms can create shorter schedule periods in terms of arbitrary \( \rho \) and \( \delta \).

![Figure 7: The scheduling lengths for \( N = 32 \) as a function of the number of slots necessary to tune transmitters.](image)

![Figure 8: The scheduling lengths for \( N = 32 \) as a function of the number of slots necessary to tune transmitters, where \( \rho \) is even.](image)
6 Conclusion

This paper concerns with the design of multi-hop packet transmissions for the AAB problem in a WDM optical star network. We partition $N$ nodes into $x$ groups and reduce the size of groups to either $\frac{\sqrt{x}}{2}$ or $\frac{x^{\frac{3}{2}}}{2}$. To construct logical topology problem can be transferred to the Hamiltonian cycles problem if $x$ is odd; otherwise it can be transferred to the round robin tournament problem. By reducing the number of designed logical topologies, we minimize the schedule length of multi-hop scheduling and decrease the expensive cost of tunable transmitters. However, the number of logical topologies is still not close to the lower bound $\left\lceil \frac{N-1}{\rho} \right\rceil$. The future work will be to keep on finding other properties for solving the AAB problem or to propose new logical topologies to create near optimal multi-hop scheduling.

References


