Construction for Strongly k-Hamiltonian Graphs *

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Abstract

The k-Hamiltonian graphs have been studied by many researchers. In this paper, we introduce strongly k-Hamiltonian graphs. A strongly k-Hamiltonian graph is also k-Hamiltonian. Moreover, we present two construction schemes for strongly k-Hamiltonian graphs including $(k+2)$-join and Cartesian product with K_2. Applying these schemes, we can construct many new strongly k-Hamiltonian graphs.
Keywords: k-Hamiltonian, strongly k-Hamiltonian graphs, $(k+2)$-join, Cartesian product, interconnection network, fault tolerance.

1 Introduction

An interconnection network is usually represented by a graph. Let $G = (V, E)$ be an undirected *graph*, where V is the *node set* and E is the *edge set* of G. The *degree* of vertex v in G, denoted by $d_G(v)$, is the number of edges incident to v. A *path* is a sequence of nodes in which every two consecutive nodes are adjacent. A *Hamiltonian path* of G is a path V if every node of V is exactly visited once. A graph is *Hamiltonian connected* if there exist Hamiltonian paths between every pair of its nodes. A *cycle* is a path if the first node and the last node are the same node. A *Hamiltonian cycle* is a cycle which traverses every node of V exactly once. A *Hamiltonian graph* is a graph that contains a Hamiltonian cycle.

Fault tolerance is desirable in the design of interconnection networks. The faults of a network corresponds to removing edges and/or nodes from the graph. Let $F \subseteq (V \cup E)$ denote the faults of graph G . Let $G - F$ denote the subgraph induced by $V - F$ of G and then deleted the edges in F from this induced subgraph. A graph G is a *k-Hamiltonian graph* if $G - F$ is Hamiltonian for all $F \subset (V \cup E)$ and $|F| = k$. In this paper, we introduce a new concept, called strongly k-Hamiltonian graphs, for the fault tolerance of Hamiltonian graphs. A graph G is a *strongly k-Hamiltonian graph* if $\forall v \in (V - F)$ there exist $k + 2 - |F|$ edges incident to v such that every pair of these edges is on some Hamiltonian cycle of $G - F$, for all $F \subseteq (V \cup E)$ and $|F| \leq k$. Thus, a strongly k-Hamiltonian graph is a k-Hamiltonian graph.

Some previous results [1, 2, 6, 10, 13, 14] have been focused on the case for $k = 1$. In [14], Wang et al. presented a construction scheme, called 3-join. Many other 3-regular 1-Hamiltonian graphs [1, 2, 6, 10, 13] can be constructed by 3-join. Some researchers studied the properties of k-Hamiltonian graphs. Many graphs have been showed to be k-Hamiltonian graphs, including recursive circulant graphs [12], crossed cubes [3], twisted cubes [5], Möbius cubes [4], pancake graphs [8]. In [7], Hung et al. presented a construction scheme, called node expansion. A family of k-Hamiltonian graphs can be constructed with node expansion, for $k \geq 1$. In this paper, we propose a construction scheme, called by $(k + 2)$-join, for strongly k-Hamiltonian graphs. The $(k + 2)$-join is the generation of both 3-join [14] and node expansion [7]. Moreover, we also present another construction scheme by Cartesian product for strongly k-Hamiltonian graphs.

The rest of this paper is organized as follows. In Section 2, we introduce the $(k+2)$-join operation. The construction scheme by Cartesian product is presented in Section 3. Section 4 includes the concluding remarks and future discussions.

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2 Strongly k-Hamiltonian Graphs and $(k+2)$-Join Operation

In this section, we will introduce the $(k+2)$-join operation and show that the $(k+2)$-join of two $(k+2)$-regular and strongly k-Hamiltonian graphs is also strongly k-Hamiltonian.

Let x be a node of the $(k+2)$-regular graph $G = (V_g, E_g)$ and y be a node of the $(k+2)$-regular graph $H = (V_h, E_h)$ with $V_g \cap V_h = \emptyset$. Let $N(x) = \{x_1, x_2, \ldots, x_{k+2}\}$ be the neighbor of x and $N(y) = \{y_1, y_2, \ldots, y_{k+2}\}$ be the neighbor of y. The $(k+2)$-*join* of G and H, denoted by $G \sqcup H$, at x and y is described as follows.

$V(G \sqcup H) = V_g \cup V_h - \{x, y\}$, and

$E(G \sqcup H) = E_g \cup E_h \cup \{(x_i, y_i) | 1 \leq i \leq k+2\} - (\{(x, x_i) | x_i \in N(x)\} \cup \{(y, y_i) | y_i \in N(y)\})$.

The graphs $(G \sqcup H)$, G and H are illustated in Figure 1. The $(k+2)$-join of two $(k+2)$-regular graphs is still $(k+2)$-regular.

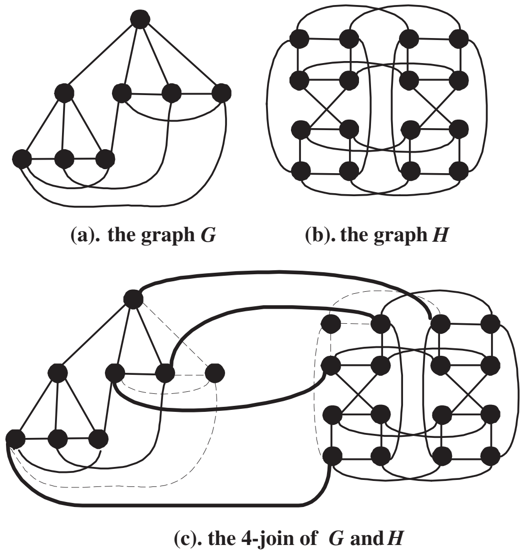

Figure 1: $(k+2)$-join

The operation 3-join proposed in [14] is the special case of $(k+2)$-join with $k = 1$. And the operation node expansion, defined in [7], is the special case of $(k+2)$-join where the graph H is a complete graph K_{k+3}. Thus the graph constructed with 3-join or node expansion can be constructed with $(k+2)$-join.

In the follows, we will show the complete graph K_n is a strongly $(n-3)$-Hamiltonian graph.

Let $K_n = (V, E)$ be the complete graph. The following theorem is proved by Ore [11].

Theorem 1 *[11] For $F \subset E$, the graph $K_n - F$ is Hamiltonian if $|F| \leq n-3$, and Hamiltonian connected if $|F| \leq n-4$.*

Applying Theorem 1, we can obtain the following lemma.

Lemma 1 *The complete graph K_n is strongly $(n-3)$-Hamiltonian for $n \geq 4$.*

Proof. We will prove this lemma by induction on n. Since K_4 is 3-regular and 1-Hamiltonian, K_4 is strongly 1-Hamiltonian. Assume the complete graph K_t is strongly $(t-3)$-Hamiltonian for $4 \leq t \leq n-1$.

Let F be the faults of $K_n = (V, E)$ with $|F| = f \leq n-3$. First, we consider that $|F \cap V| = i > 0$. Thus the graph $K_n - F$ is isomorphic to $K_{n-i} - F'$ for some $F' \subset E$ and $|F'| \leq f - i$. By induction hypotheses, the graph $K_{n-i} - F'$ is strongly $(n-i-3)$-Hamiltonian. Hence, $\forall v \in (V(K_{n-i}) - F')$ there exist $n-1-f$ edges incident to v such that every pair of these edges is on some Hamiltonian cycle of $K_{n-i} - F'$ for all $F' \subseteq E$ and $|F'| \leq n-i-3$. Therefore, $\forall v \in (V(K_n) - F)$ there exist $n-1-f$ edges incident to v such that every pair of these edges is on some Hamiltonian cycle of $K_n - F$ for all $F \subseteq (V \cup E)$ and $|F| \leq n-3$. Thus, K_n is strongly $(n-3)$-Hamiltonian.

Next, we consider that $F \subset E$. Applying **Theorem 1**, we can obtain that when $f = n-3$, $K_n - F$ is Hamiltonian. This satisfies the conditions of strongly $(n-3)$-Hamiltonian graphs with $f = n-3$. Now we consider that $f \leq n-4$. Let $E(u)$ be the set of edges incident to u, for $u \in V$. Since $f \leq n-4$, if $\forall u \in V |E(u) \cap F| \neq 1$ then there exists two nodes u_1 and u_2 such that $|E(u_1) \cap F| = |E(u_2) \cap F| = 0$. Let G_v be the subgraph of K_n induced by $V - \{v\}$. $F_{G_v} = F \cap (V(G_v) \cup E(G_v))$. Thus G_v is isomorphic to $K_n - 1$. By induction hypotheses, G_v is strongly $(n-4)$-Hamiltonian. We will show K_n is strongly $(n-3)$-Hamiltonian for $f \leq n-4$ by the following cases.

- **Case 1**: $\exists u, |E(u) \cap F| = 1$ $|F_{G_u}| = |F| - 1 \leq n-5$. Thus, $\forall v \in (V(G_u) - F_{G_u})$ there exist $n-2-|F_{G_u}| = n-1-|F|$ edges such that every pair of these edges is on some Hamiltonian cycle of $G_u - F_{G_u}$. These Hamiltonian cycles can be extended to Hamiltonian cycle of $K_n - F$ since $|E(u) \cap F| = 1$. Applying **Theorem 1**, we can obtain that $G_u - F_{G_u}$ is Hamiltonian connected. Hence, every pair of $E(u)$ is on some Hamiltonian cycle of $K_n - F$. Therefore, $\forall v \in (V - F)$ there exist $n-1-|F|$ edges such that every pair of these edges is on some Hamiltonian cycle of $K_n - F$. Thus K_n is strongly k-Hamiltonian.

- **Case 2**: $\exists u_1, u_2$, $|E(u_1) \cap F| = |E(u_2) \cap F| = 0$. For $v \in (V(G_{u_1}) - F_{G_{u_1}})$ there exist $n - 2 - |F_{G_{u_1}}| = n - 2 - f$ edges, denoted by $E_1(v)$ incident to v such that every pairs of $E_1(v)$ is on some Hamiltonian cycle of $G_{u_1} - F_{G_{u_1}}$. Since $|E(u_1) \cap F| = 0$, $(u, v) \notin F$ for all $v\ inV$. We can construct the Hamiltonian cycles of $K_n - F$ by replacing some edge (a, b) of previous Hamiltonian cycles by (a, u) and (u, b). Thus, $\forall v \in (V - F - \{u_1\})$ every pair of edges in $(E_1(v) \cup \{(u_1, v)\})$ is on some Hamiltonian cycle of $K_n - F$. Similarly, $\forall v \in (V - F - \{u_2\})$, every pair of edges in $(E_1(v) \cup \{(u_2, v)\})$ is on some Hamiltonian cycle of $K_n - F$. Therefore, $\forall v \in (V - F)$, there exist $n - 1 - f$ edges such that every pair of these edges is on some Hamiltonian cycle of $K_n - F$. Thus, K_n is strongly $(n-3)$-Hamiltonian.

  This lemma is follows. $\square$

In the following theorem, we will show that the $(k + 2)$-join of two $(k + 2)$-regular and strongly k-Hamiltonian graphs is also strongly k-Hamiltonian.

Let F, F_g and F_h denote sets of faulty components including faulty nodes and edges in $(k+2)$-join, G and H, respectively. And let $F_{gh} = \{(x_i, y_i)| 1 \leq i \leq k + 2\}$. Thus $F = F_g \cup F_h \cup F_{gh}$.

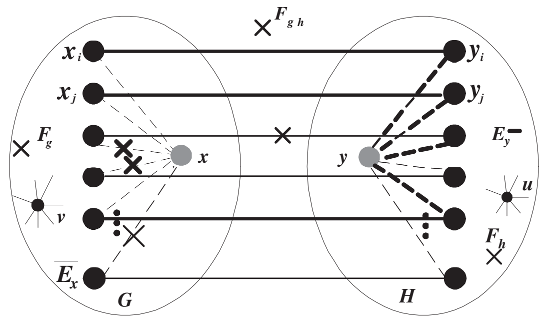

Figure 2: The strongly k-Hamiltonian of $(G \sqcup H) - F$.

Theorem 2 *If G and H are $(k + 2)$-regular and strongly k-Hamiltonian graphs, then $G \sqcup H$ is also a strongly k-Hamiltonian graph.*

Proof. We will prove that $\forall v \in (V(G \sqcup H) - F)$, there exist $(k + 2) - |F|$ edges incident to v (denoted by E_v), such that every pair of E_v is on some Hamiltonian cycle of $(G \sqcup H) - F$, $|F| \leq k$. Let $G \sqcup H$ be the $(k+2)$-join of G and H at x and y, respectively. Thus, let $\overline{E_x} = \{(x, x_i)|(y, y_i) \notin E_y$, or $(x_i, y_i) \in F_{gh}\}$. $|\overline{E_x}| = |F_h| + |F_{gh}|$. We delete every edge in $\overline{E_x}$ of G. Thus the faults of G is at most $|F_g| + |F_h| + |F_{gh}| = F$. Since G is strongly k-Hamiltonian, $\forall v \in (V_g - F_g)$, there exist $k + 2 - |F|$ edges such that every pair of these edges is on some Hamiltonian cycle of $G - F_g - \overline{E_x}$. Let HC_g be an arbitrary Hamiltonian cycle of $G - F_g - \overline{E_x}$. Thus, HC_g contains two edges incident to x, (x, x_i) and (x, x_j) for some $y_i, y_j \in E_y$. Since $(y, y_i), (y, y_j) \in E_y$, there exists a Hamiltonian cycle HC_h of $H - F_h$ which contains (y, y_i) and (y, y_j). Hence, $HC_g \cup HC_h - \{x, y\} \cup \{(x_i, y_i), (x_j, y_j)\}$ forms a Hamiltonian cycle of $(G \sqcup H) - F - \overline{E_x}$. This construction is illustrated in Figure 2. Thus, $\forall v \in (V_g - F_g)$ there exist $k + 2 - |F|$ edges incident to v, such that every pair of these edges is on some Hamiltonian cycle of $(G \sqcup H) - F$. With the similar construction, for all vertices $u \in V_h - F_h$, there exist $k + 2 - |F|$ edges incident to u, such that every pair of these edges is on some Hamiltonian cycle of $(G \sqcup H) - F$. Therefore, $G \sqcup H$ is strongly k-Hamiltonian. $\square$

Applying the **Lemma 1** and **Theorem 2**, we can obtain that the graphs constructed in [7] are all strongly k-Hamiltonian. On the other hand, the graphs constructed only by **Lemma 1** and **Theorem 2** also can be constructed by node expansion in [7]. We need some other construction schemes to produce new strongly k-Hamiltonian graphs.

3 Strongly k-Hamiltonian Graphs and Cartesian Product Operation

In this section, we will show that the Cartesian product of a $(k + 2)$-regular, strongly k-Hamiltonian graph and K_2 is strongly $(k + 1)$-Hamiltonian.

The graph $G \times H$ is the Cartesian product of the graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$. The node set $V(G \times H)$ is $\{ (u, v)| \forall u \in V_1$ and $v \in V_2$ $\}$. The edge set $E(G \times H)$ is $\{((u_1, v_1), (u_2, v_2))|$ if $((u_1, u_2) \in E_1$ and $v_1 = v_2)$ or $(u_1 = u_2$, and $(v_1, v_2) \in E_2)\}$. Thus, $V = V(G \times K_2) = \{(v, i)| \forall v \in V(G)$ and $i = 1, 2\}$ and $E = E(G \times K_2) = \{((u, i), (v, j))|$ if $((u, v) \in E(G)$ and $i = j)$

or $u = v$ and $i \neq j\}$. Let G_i be a sub-graph of $(G \times K_2)$ and induced by vertices set $\{(v, i)\}$ for $i = 1, 2$. To be specific, the node set of G_i is $\{(v, i)|\forall v \in V\}$ and the edge set of G_i is $\{((u, i), (v, i))|\forall (u, v) \in E\}$.

Let F, F_1 and F_2 denote sets of faulty components including faulty nodes and edges in $(G \times K_2)$, G_1 and G_2, respectively. Let F_3 be the set of faulty edges between G_1 and G_2. That is, $F_3 = F \cap \{((u, 1), (u, 2))|\forall u \in V\}$. Thus $F = F_1 \cup F_2 \cup F_3$.

Theorem 3 *If G is $(k+2)$-regular and strongly k-Hamiltonian, $G \times K_2$ is $(k+3)$-regular and strongly $(k+1)$-Hamiltonian.*

Proof. We will show this theorem with the following cases.

- **Case 1**: $|F| = k + 1$

  We will prove that for all $(v, i) \in (V - F)$ in $G \times K_2$, there exist $(k + 3) - |F| = 2$ edges incident to (v, i), such that these two edges are on a Hamiltonian cycle of $G \times K_2 - F$. Thus, we only need to show that there exists a Hamiltonian cycle in $G \times K_2 - F$.

  Since G is strongly k-Hamiltonian, it is also k-Hamiltonian. In [9] C.N. Hung proved that if G is $(k+2)$-regular and k-Hamiltonian then $G \times K_2$ is $(k + 1)$-Hamiltonian. Thus, there exists a Hamiltonian cycle in $G \times K_2 - F$ when $|F| = k + 1$.

- **Case 2**: $|F| \leq k$ and ($F = F_1$ or $F = F_2$). Without loss of generality, we can assume that $F = F_1$.

  We will prove that for all $(v, i) \in (V - F)$, there are $k + 3 - |F_1|$ edges incident to (v, i), such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$.

- **Case 2.1**: Consider the nodes of G_1.

  Let $(a, 1)$ be an arbitrary node of G_1. Since G_1 is strongly k-Hamiltonian, there are $k+2-|F_1|$ edges incident to $(a, 1)$, denoted by E_{a_1} and every pair of E_{a_1} is on some Hamiltonian cycle of $G_1 - F_1$. Let HC_1 be an arbitrary Hamiltonian cycle of $G_1 - F_1$ contains two edges of E_{a_1}. Let $(b, 1) \neq (a, 1)$ be a fault-free node. Thus, HC_1 contains two edges incident to $(b, 1)$. At least one edge of these two edges denoted by $((b, 1), (c, 1))$, is not incident to $(a, 1)$. Since $|F_2| = 0$, the edge $((b, 2), (c, 2))$ is on some Hamiltonian cycle HC_2 of G_2. Thus, $HC_1 \cup HC_2 - \{((b, 1), (c, 1)), ((b, 2), (c, 2))\} \cup \{((b, 1), (b, 2)), ((c, 1), (c, 2))\}$ forms a Hamiltonian cycle of $G \times K_2 - F$. Hence, there exist $|E_{a_1}|$ edges incident to an arbitrary node $(a, 1) \in V(G_1)$, such that every pair of E_{a_1} is on some Hamiltonian cycle of $G \times K_2 - F$. Additionally, HC_1 also contains two edges incident to $(a, 1)$, denoted by $((a, 1), (d, 1))$ and $((a, 1), (e, 1))$. The edges $((a, 2), (d, 2))$ and $((a, 2), (e, 2))$ are on some Hamiltonian cycle HC_3 and HC_4, respectively of G_2. Thus, $HC_1 \cup HC_3 - \{((a, 1), (d, 1)), ((a, 2), (d, 2))\} \cup \{((a, 1), (a, 2)), ((d, 1), (d, 2))\}$ and $HC_1 \cup HC_4 - \{((a, 1), (e, 1)), ((a, 2), (e, 2))\} \cup \{((a, 1), (a, 2)), ((e, 1), (e, 2))\}$ are both Hamiltonian cycles of $G \times K_2 - F$. Figure 3 is illustrated for this case. Hence, every pair of $E_{a_1} \cup \{((a, 1), (a, 2))\}$ is on some Hamiltonian cycle of $G \times K_2 - F$. Therefore, there exist $k+3-|F|$ edges incident to $(a, 1)$, for all $(a, 1) \in V_1 - F$, such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$.

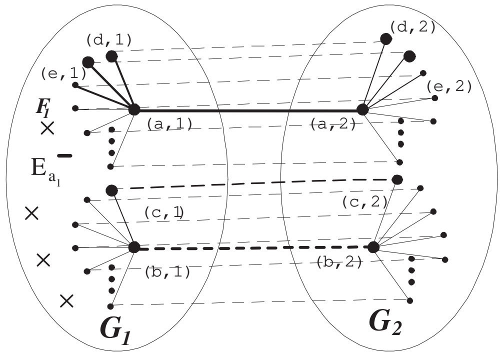

Figure 3: For all $(a, 1) \in (V_1 - F)$, there exist $k+3-|F|$ edges incident to $(a, 1)$ such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$.

- **Case 2.2**: Consider the nodes in G_2.

  Let $(a, 2)$ be an arbitrary node of G_2. Let $(b, 1)$ be a fault-free node of G_1 and $(b, 2) \neq (a, 2)$. Since G_1 is strongly k-Hamiltonian, there exist $k + 2 - |F_1|$ edges incident to $(b, 1)$ (denoted by E_{b_1}) such that every pair of E_{b_1} is on some Hamiltonian cycle of $G_1 - F_1$. Let $\overline{E_{b_2}}$ be the edge set $\{((b, 2), (v, 2))|((b, 1), (v, 1)) \notin E_{b_1}\}$. Thus, $|\overline{E_{b_2}}| = |F_1|$. We delete $|\overline{E_{b_2}}| - 1$ edges of $\overline{E_{b_2}}$ (denoted by $\overline{E_{b_2}}'$). Since G_2 is strongly

k-Hamiltonian, there exist $k+2-(|\overline{E_{b_2}}|-1) = k+3-|F|$ edges incident to $(a,2)$ such that every pair of these edges is on some Hamiltonian cycle HC_2 of $G_2 - \overline{E_{b_2}}'$. Thus, HC_2 contains at least one edge $((b,2),(c,2))$ incident to $(b,2)$ where $((b,1),(c,1)) \in E_{b_1}$. Since $((b,1),(c,1)) \in E_{b_1}$, $((b,1),(c,1))$ is on some Hamiltonian cycle HC_1 of $G_1 - F_1$. Hence, $HC_1 \cup HC_2 - \{((b,1),(c,1)),((b,2),(c,2))\} \cup \{((b,1),(b,2)),((c,1),(c,2))\}$ forms a Hamiltonian cycle of $G \times K_2 - F$. Figure 4 is illustrated for this case. Therefore, $\forall (a,2) \in V(G_2)$ there exist $k+3-|F|$ edges incident to $(a,2)$, such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$.

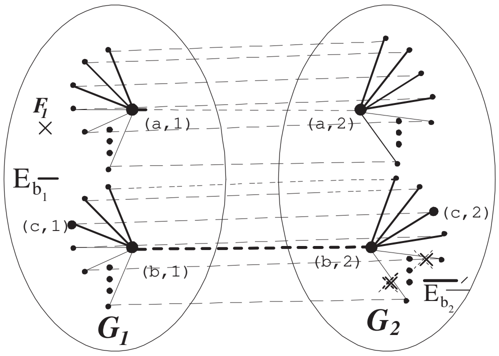

Figure 4: For all $(a,2) \in V(G_2)$, there exist $k+3-|F|$ edges incident to $(a,2)$ such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$.

- **Case 3**: $|F| > |F_1|$ and $|F| > |F_2|$ and $|F| \le k$.

  We will prove that $\forall (v,i) \in (V-F)$, there exist $k+3-|F| = k+3-|F_1|-|F_2|-|F_3|$ edges incident to (v,i), $\forall (v,i) \in (V-F)$, such that every pair of these edges are on some Hamiltonian cycle of $G \times K_2 - F$.

  Let $(a,1),(b,1),(b,2)$ be fault-free nodes and $((b,1),(b,2))$ be a fault-free edge. Since G_2 is strongly k-Hamiltonian, there are $k+2-|F_2|$ edges incident to $(b,2)$ (denoted by E_{b_2}) such that every pair of E_{b_2} is on some Hamiltonian cycle of $G_2 - F_2$. Let $\overline{E_{b_1}}$ be the edge set $\{((b,1),(v,1)) | ((b,2),(v,2)) \notin E_{b_2}$ or $((v,1),(v,2)) \in F_3\}$. Thus, $|\overline{E_{b_1}}| \le |F_2| + |F_3|$. We delete $|\overline{E_{b_1}}|-1$ edges of $\overline{E_{b_1}}$ (denoted by $\overline{E_{b_1}}'$). Since G_1 is strongly k-Hamiltonian, $\forall (a,1) \in V(G_1)$ there are $k+2-|F_1|-|\overline{E_{b_1}}'| \ge k+3-|F|$ edges incident to $(a,1)$, such that every pair of these edges is on some Hamiltonian cycle of $G_1 - F_1 - \overline{E_{b_1}}'$. Let HC_1 be an arbitrary Hamiltonian cycle contains a pair of these $k+3-|F|$ edges incident to $(a,1)$ of $G_1 - F_1 - \overline{E_{b_1}}'$. There exists a edge $((b,1),(c,1))$ of HC_1 where $((b,2),(c,2)) \in E_{b_2}$. Since the edge $((b,2),(c,2)) \in E_{b_2}$, it is on some Hamiltonian cycle HC_2 of $G_2 - F_2$. Hence, $HC_1 \cup HC_2 - \{((b,1),(c,1)),((b,2),(c,2))\} \cup \{((b,1),(b,2)),((c,1),(c,2))\}$ forms a Hamiltonian cycle of $G \times K_2 - F$. Thereby, $\forall (a,1) \in (V(G_1) - F_1)$ there are $k+3-|F|$ edges incident to $(a,1)$, such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$. Figure 5 is illustrated for this case. Applying the similar construction scheme, we can obtain that $\forall (a,2) \in (V(G_2) - F_2)$ there are $k+3-|F|$ edges incident to $(a,2)$, such that every pair of these edges is on some Hamiltonian cycle of $G \times K_2 - F$. Therefore, $G \times K_2$ is strongly $(k+1)$-Hamiltonian. $\square$

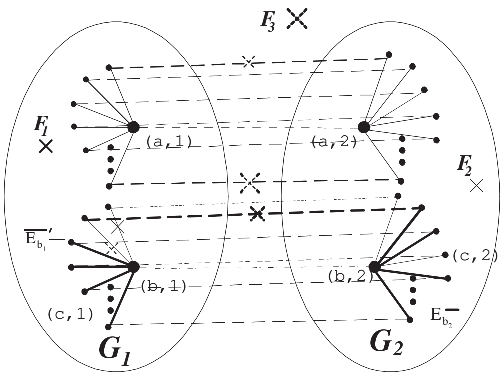

Figure 5: The case for $|F| > |F_1|$, $|F| > |F_2|$ and $|F| \le k$ in $G \times K_2$

Applying the **Lemma 1**, **Theorem 2** and **Theorem 3**, we can construct a family of strongly k-Hamiltonian graphs. Moreover, these graphs are also new k-Hamiltonian graphs. The graphs $K_4 \times K_2$, K_5 and $(K_4 \times K_2) \sqcup K_5$ are strongly 2-Hamiltonian as illustrated in Figure 6.

4 Conclusion

In this paper, we introduce the concept of strongly k-Hamiltonian graphs. We also present

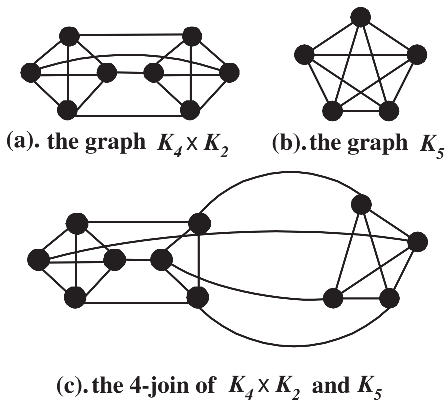

Figure 6: Some strongly k-Hamiltonian graphs

two construction schemes, the $(k + 2)$-join and Cartesian product with K_2, for strongly k-Hamiltonian graphs. Applying these schemes, we can construct a series of $(k + 2)$-regular and strongly k-Hamiltonian graphs. In the future, we hope to present more construction schemes for the strongly k-Hamiltonian graphs.

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