Dept. of Computer Science and Engineering, undergraduate
National Sun Yat-sen University
Advanced Programming and Practice - Final Exam., June 23, 2011, 9:10–12:00

1. Explain each of the following terms. (20%)
   (a) *topological ordering*
   (b) *sum of subset problem*
   (c) *hill climbing method*
   (d) *transitive closure of a graph*
   (e) *closest pair problem*

2. Given an undirected graph, what are the *Euler cycle* and *Hamiltonian cycle* problems? Please point out their main difference. (10%)

3. (a) Give an algorithm for solving the shortest path (from single source) problem of an undirected graph. Give an example to illustrate your algorithm. (10%)
   (b) Please derive the time complexity of your algorithm. (5%)

4. (a) What is the definition of the *2D ranking problem*? Give an example to illustrate your explanation. (5%)
   (b) Give a divide-and-conquer algorithm for solving the problem. Please derive the time complexity of your algorithm. (10%)

5. (a) Give the definition of the *longest common subsequence (LCS)* problem. (5%)
   (b) Design an algorithm to solve the LCS (length) problem. (5%)

6. Let $S_n = \{1, 2, \cdots, n\}$. An m-combination of S_n is obtained by selecting m distinct integers out of the n integers and it is represented with *lexicographic order*. For example, both combinations (2 4 1) and (4 1 2) are the same, and they are represented with (1 2 4). Now let $x = (x_1\ x_2\ \cdots\ x_m)$ and $y = (y_1\ y_2\ \cdots\ y_m)$ be two m-combinations of S_n. We say that x precedes y in *lexicographic order* if there exists an i, $1 \leq i \leq m$, such that $x_j = y_j$ for all $j < i$ and $x_i < y_i$. And the *rank* of an m-combination c, denoted as $r(c)$, is the number of combinations before c in the lexicographical order. For example, all 3-combinations of S_4 in lexicographical order are (1 2 3), (1 2 4), (1 3 4), (2 3 4). Thus $r(1\ 2\ 3) = 0$, $r(1\ 2\ 4) = 1$, $r(1\ 3\ 4) = 2$, and $r(2\ 3\ 4) = 3$. Answer the following questions for the 5-combinations of S_{10}.
   (a) What is the next one of (2 3 5 9 10)? (5%)

(b) What is $r(3\ 4\ 6\ 8\ 9)$? Explain how do you calculate? (5%)

(c) Which combination has the rank 178? Explain how do you calculate? (5%)

7. Given a set S of numbers, you are asked to find whether there exist four numbers a, b, c, $d \in S$ such that $a+b+c=d$. For solving this problem, a trivial method is to examine all 4-number combinations, which requires $O(n^4)$ time where $|S|=n$. However, this method is not efficient enough. Please design a more efficient algorithm to solve this problem and analyze the time complexity. Note that if your algorithm needs less than $O(n^3 \log n)$ time, you will get higher score. (15%)