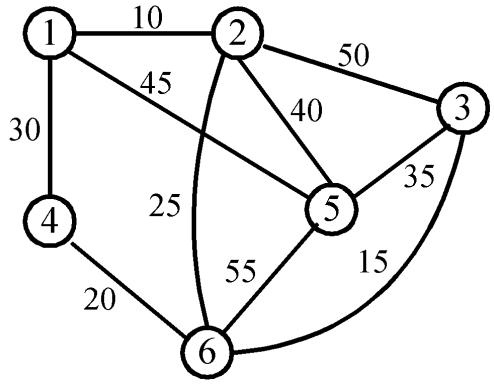
Department of Computer Science and Engineering
National Sun Yat-sen University
Advanced Programming and Practice - Final Exam., June 25, 2015

1. Explain the *Eulerian cycle* of an undirected graph. What is its sufficient and necessary condition? (10%)

2. (a) Please present an algorithm for constructing the *minimum spanning tree* of a given connected, undirected and weighted graph. Explain your algorithm with the following graph. (10%)
   (b) Analyze the time complexity of your algorithm. (5%)



3. (a) What is the *knapsack* problem? (5%)
   (b) Design a *greedy* algorithm for solving the above problem. (10%)

4. (a) Given n points on the 2D plane, what is the *2D maxima* problem? Please give an example to describe your answer. (5%)
   (b) Present a *divide-and-conquer* algorithm for solving the above problem. And analyze the time complexity of your algorithm. (10%)

5. Explain the *breadth-first search* method in a graph. What data structure should be used? What is the time complexity? (10%)

6. What is the *hill climbing* method for searching the solution of a given problem? (5%)

7. Let S_n={1,2,…,n}. An m-combination of S_n is obtained by selecting m distinct integers out of the n integers and it is represented with *lexicographic order*. For example, the two combinations (2,4,1) and (4,1,2) are the same, and they are represented with (1,2,4). Now let x=(x_1, x_2,…, x_m) and y=(y_1, y_2, y_m) be two m-combinations of S_n . We say that x precedes y in l*exicographic order* if there exists an i, $1 \le i \le m$, such that x_j=y_j for all j<i and $x_i < y_i$. And the *rank* of an m-combination c, denoted as $r(c)$, is the number of combinations before c in the lexicographical order. For example, all 3-combinations of S_4 in lexicographical order are (1, 2, 3), (1,2,4), (1,3,4), (2,3,4). Thus r(1,2,3)=0, r(1,2,4)=1, r(1,3,4)=2, and r(2,3,4)=3. Answer the following questions for the 5-combinations of S_{10}.
   (a) What is the next one of (2,3,5,9,10)? (5%)

(b) What is $r(3,4,6,8,9)$? Explain how do you calculate? (5%)
(c) Which combination has the rank 178? Explain how do you calculate? (5%)

8. In the *interval cover* problem, we are given a set of integer intervals $T=\{I_1=[s_1,e_1], I_2=[s_2,e_2], \ldots, I_m=[s_m,e_m]\}$, where each interval $[s_i,e_i]$, $s_i \le e_i$, covers the segment starting from position s_i and ending at position f_i. The problem is to ask the minimum number of selected intervals for exactly covering the line segment from position 1 to position n, where every two selected intervals are disjoint. For example, $n=10$, $T=\{I_1=[1,2], I_2=[1,3], I_3=[2,4], I_4=[2,6], I_5=[3,4], I_6=[4,7], I_7=[6,10], I_8=[7,10], I_9=[2,3]\}$. The answer of the problem is 3, which contains I_1, I_4, and I_7. Please design an algorithm for solving this problem and analyze the time complexity of your algorithm. Note that the time complexity of your algorithm should be a polynomial function of n. (Hints: Remember how to solve the optimal binary search tree problem and the matrix multiplication chain problem.) (15%)

Answer:

7 (a) (2,3,6,7,8) (b) 209 (c) (2,4,7,8,10)