Dept. of Computer Science and Engineering, Graduate
National Sun Yat-sen University
Design and Analysis of Algorithms - Final Exam., Jan. 14, 2014

1. Multiple choices (There may be zero or more correct answers. If there is no correct answer, you should write down "None".) (24%)
   Answer: (a) ABD (b) AB (c) BCD (d) ACD (e) B (f) BD

   (a) Which statement(s) is correct? (A) If problem A is *NP-hard* and it is an NP problem, then A is NP-complete. (B) If problem A is *NP-complete*, then A is *NP-hard* and it is an NP problem. (C) If problem A is *NP-complete*, then A has no polynomial time algorithm in the worst case. (D) If problem A is a P problem, then A is an NP problem.

   (b) Which statement(s) is correct? (A) If problem A is *NP-complete*, then all NP problems polynomially reduce to A. (B) If both problems A and B are *NP-complete*, then A polynomially reduces to B and B polynomially reduces to A. (C) If problem A is an NP problem and A polynomially reduces to problem B, then B is *NP-complete*. (D) If problem A is *NP-complete* and B polynomially reduces to problem A, then B is NP-complete.

   (c) Let $f(n) = \log \frac{n}{1} + \log \frac{n}{2} + \log \frac{n}{3} + \cdots + \log \frac{n}{n}$. Then, (A) $f(n) = O(\log^2 n)$. (B) $f(n) = O(n \log n)$. (C) $f(n) = O(n \log^2 n)$. (D) $f(n) = O(n^2)$.

   (d) Let A denote the *knapsack* problem and B denote the *0/1 knapsack problem*. (A) If A and B have the same input, then the solution of B is no more than that of A. (B) A is *NP-hard*. (C) B is *NP-hard*. (D) The decision version of B is an NP problem.

   (e) Given a set S of n 2D plain points, which statement(s) is correct for the *1-center* problem? (A) At least three points locate on the curve of the smallest circle containing the n points. (B) For the *constrained 1-center* problem, it is possible that only one point locates on the curve of the smallest circle containing the n points. (C) The center (solution) can be obtained by averaging the positions of the n points. (D) If all of the n points locate on the x-axis, then the center (solution) is equal to the median of the n points.

   (f) Which statement(s) is correct for an undirected graph? (A) The length of an *Eulerian cycle* is no more than that of a *Hamiltonian cycle* if these two cycles exist. (B) The number of nodes with odd degrees is even. (C) The *bottleneck traveling salesperson* problem is to minimize the total length of the solution. (D)

If a graph is *bi-connected*, then there exists at least one path between every pair of nodes.

2. y is a *quadratic residue* mod x if $z^2 = y \bmod x$ for some z, $0 < z < x$, $GCD(x, z) = 1$ and $GCD(x, y) = 1$. Then define $QR(x) = \{(x, y) | y \text{ is a quadratic residue mod } x\}$. Please give $QR(9)$. (9%)
   answer: (9,1), (9,4), (9,7)

3. Give an algorithm for solving the single source (to all other nodes) shortest path problem of a connected undirected graph. Analyze the time complexity of your algorithm. (10%)

4. Given a set of n 2D points with the *Voronoi diagram*, please design an algorithm for constructing the *convex hull* with $O(n)$ time. Analyze the time complexity of your algorithm. (10%)

5. In the searching strategy, explain *breadth-first search*, *depth-first search*, *best-first search* and *hill climbing*. (8%)

6. In the self-organizing sequential search heuristics, what are the *transpose heuristics, move-to-front heuristics* and *count heuristics*? (12%)

7. Prove that the *partition* decision problem polynomially reduces to the *bin packing* decision problem. (12%)

8. Given a set S of n positive integers, the 4-subset problem is to determine whether there exist four distinct elements a, b, c and d in S such that $a + b + c = d$. Please design an algorithm to solve this problem, and analyze the time complexity of your algorithm. (15%)

   (a) If the time complexity of your algorithm is $\mathrm{O}(n^3 \log n)$, then you will get only 5% points.

   (b) If the time complexity of your algorithm is $\mathrm{O}(n^2 \log n)$, then you will get the full 15% points.