Department of Computer Science and Engineering
National Sun Yat-sen University
Design and Analysis of Algorithms - Final Exam., Jan. 10, 2017

1. Multiple choices (There may be zero or more correct answers. If there is no correct answer, you should write down "None".) (16%)

   (a) Which statement(s) is correct? (A) If problem A is NP-hard, then A is NP-complete. (B) If problem A is NP-complete, then A is NP-hard. (C) The satisfiability problem is NP-hard. (D) If problem A is an NP problem, and A polynomially reduces to another problem B, then B is NP-hard.

   (b) Which statement(s) is correct? In the following, let n denote the number of input elements. (A) The time complexity of quicksort is $O(n\log n)$. (B) The best case for insertion sort is that the input elements are already sorted. (C) Heapsort requires $O(n^2)$ time in the worst case. (D) Heapsort is an optimal sorting algorithm in the average case.

   (c) Which statement(s) is correct for the minimum spanning tree (MST) of a connected graph $G = (V, E)$? (A) In the MST, for any two nodes u and v, there exists exactly one path connecting them. (B) The MST has exactly $|V| - 1$ edges. (C) In the MST, each tree edge has cost no more than each non-tree edge. (D) In the MST, if node v is of degree 1 and (u,v) is a tree edge, then the weight of (u,v) is the minimum among all $(x,v) \in E$.

   (d) Given a graph $G = (V, E)$, let $Q = (V_Q, E_Q)$ be a (maximal) clique of G. Which statement(s) is correct? (A) $V_Q = V$ and $E_Q \subseteq E$. (B) $V_Q \subseteq V$ and $E_Q = E$. (C) For each vertex $u \in V - V_Q$, there exists one vertex $v \in V_Q$ such that $(u,v) \notin E$. (D) There exists one vertex $u \in V - V_Q$ such that for all $v \in V_Q$, $(u,v) \notin E$.

2. Derive $T(n)=2T(n/2)+n$, for $n\geq 2$, and $T(1)=1$. (10%)

3. Please give the definitions of (a) convex problem; (b) one-center problem; (c) rectilinear m-center problem. (12%)

4. In the searching strategy, explain breadth-first search, depth-first search, best-first search and hill climbing. (8%)

5. Use the prune-and-search approach to design an algorithm for selecting the kth smallest element among n input elements. Your algorithm should be with $O(n)$ time. (12%)

6. An approximate algorithm for solving the node cover problem of a graph $G = (V, E)$ is given as follows. Let N denote the solution (node cover). Initially, $F = E$. Arbitrarily select an edge $(u,v) \in F$, then add nodes u and v into N. Next, remove all edges incident to u or v from F. Repeat the above procedure until F becomes empty. Suppose that C is the optimal solution (node cover). Show that $|N| \leq 2|C|$. (10%)

7. Given a binary number (with n bits) $B=b_1b_2\ldots b_n$, let $f(i)=(b_ib_{i+1}\ldots b_{i+w-1}) \bmod p$, where p is a positive integer and w is the window size. Suppose $f(i)$ has been calculated and its value is r. Please derive the method to calculate $f(i+1)$ by using r. (10%)

8. Prove that the sum of subset decision problem polynomially reduces to the partition problem. (12%)

9. Given two sets A and B, each consisting of n integers, design an efficient algorithm to check whether A is equal to B or not. And analyze the time complexity of your algorithm. Note that your algorithm should be with $O(n\log n)$ time. (10%)

Answer:
1. BC, BD, ABD, C