1. Explain each of the following terms. (20%)

(a) *optimal binary search tree problem*

(b) *partition problem*

(c) *convex hull problem*

(d) *branch and bound*

(e) *closest pair problem*

2. (a) What is the definition of *minimum spanning tree*? Give an example to illustrate your answer. (5%)

(b) Give an algorithm for solving the minimum spanning tree problem. Give an example to illustrate your algorithm. (10%)

(c) Please derive the time complexity of your algorithm. (5%)

3. (a) What is the definition of *knapsack problem*? What is the difference between *knapsack problem* and *0/1 knapsack problem*? (7%)

(b) Give an algorithm for solving the knapsack problem. Please derive the time complexity of your algorithm. (8%)

4. (a) What is *depth-first search* in the solution searching strategy? (5%)

(b) What is *hill-climbing* in the solution searching strategy? (5%)

(c) What is *best-first search* in the solution searching strategy? (5%)

5. There is a rectangle brick with size 1 × 2. You may put a brick horizontally, that is, the height is 1 and the width is 2. You may also put a brick vertically, that is, the height is 2 and the width is 1. Suppose you have a long wall built with the bricks. The wall has height 2 and width n. If n = 2, then you have two ways to build the wall, either 2 horizontal bricks or 2 vertical bricks. How many ways are there for building the wall by using these bricks? How much time is required for calculating your answer? (15%)

6. A sequence \((a_1, a_2, \ldots, a_n)\) is *bitonic* if there exists \(i, 1 \leq i \leq n\), such that \(a_1 \leq a_2 \leq \cdots \leq a_i\) and \(a_i \geq a_{i+1} \geq \cdots \geq a_n\). For example, \((1,2,3,4,10,19)\) is bitonic since \(i = 6\) and \((1,7,12,13,14,10,9,5)\) is also bitonic since \(i = 5\). There is an important theorem as follows.
Given a binotic sequence $A = (a_1, a_2, \cdots, a_{2n})$, let $b_i = \min\{a_i, a_{n+i}\}$ and $c_i = \max\{a_i, a_{n+i}\}$. Then the two sequences $B = (b_1, b_2, \cdots, b_n)$ and $C = (c_1, c_2, \cdots, c_n)$ are both bitonic and $b_i \leq c_j$ for all $i$ and $j$.

Suppose that the minimum and maximum of a pair of numbers can be found with a comparison. In other words, $b_i$ and $c_i$ can be obtained with a comparison performed on $a_i$ and $a_{i+n}$.

(a) Given a bitonic sequence $A$ of length $n = 2^k$, $k$ is a positive integer, how do you sort $A$ into non-decreasing order by applying the above theorem? Design your algorithm and use the example $(1,7,12,13,14,10,9,5)$ to illustrate your algorithm. (10%)

(b) What is the time complexity of your algorithm? Give your reasoning. (5%)