The String-to-String Correction Problem

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ABSTRACT. The *string-to-string correction problem* is to determine the distance between two strings as measured by the minimum cost sequence of "edit operations" needed to change the one string into the other. The edit operations investigated allow changing one symbol of a string into another single symbol, deleting one symbol from a string, or inserting a single symbol into a string. An algorithm is presented which solves this problem in time proportional to the product of the lengths of the two strings. Possible applications are to the problems of automatic spelling correction and determining the longest subsequence of characters common to two strings.

KEY WORDS AND PHRASES: string correction, editing, string modification, correction, spelling correction, longest common subsequence

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1. *Introduction*

Morgan [1] considers four editing operations which can be applied to keypunched words in order to undo certain common keypunch errors. His paper describes a technique for finding those language tokens (usually complier key words, such as BEGIN or WRITE) which lie a distance of one edit operation away from the given, presumably incorrect, input token.

Based on three of Morgan's operations, we define a general notion of "distance" between two strings and present an algorithm for computing the distance in time proportional to the product of the lengths of the strings. The operations we consider are: (1) changing one character to another single character; (2) deleting one character from the given string; (3) inserting a single character into the given string.

This notion of edit distance and the efficient algorithm for computing it have obvious applications to problems of spelling correction and may be useful in choosing mutually distant key words in the design of a programming language. The algorithm may also be

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used, as a special case, to find the longest subsequence of characters common to two strings.

2. *Edit Distance*

Let A be a finite string (or sequence) of characters (or symbols). $A\langle i\rangle$ is the ith character of string A; $A\langle i:j\rangle$ is the ith through jth characters (inclusive) of A (so $A\langle i:j\rangle = A\langle i\rangle A\langle i+1\rangle \cdots A\langle j\rangle$), and $A\langle i:j\rangle = \Lambda$, the null string, if $i > j$. $|A|$ denotes the length (number of characters) of string A.

An *edit operation* is a pair $(a, b) \neq (\Lambda, \Lambda)$ of strings of length less than or equal to 1 and is usually written $a \rightarrow b$. String B results from the application of the operation $a \rightarrow b$ to string A, written $A \Rightarrow B$ *via* $a \rightarrow b$, if $A = \sigma a \tau$ and $B = \sigma b \tau$ for some strings σ and τ. (Readers familiar with formal language theory will note the similarity between an edit operation and a production of a grammar.) We call $a \rightarrow b$ a *change* operation if $a \neq \Lambda$ and $b \neq \Lambda$; a *delete* operation if $b = \Lambda$; and an *insert* operation if $a = \Lambda$.

Let S be a sequence $s_1, s_2, \cdots, s_m$ of edit operations (or *edit sequence* for short). An *S-derivation from A to B* is a sequence of strings $A_0, A_1, \cdots, A_m$ such that $A = A_0$, $B = A_m$, and $A_{i-1} \Rightarrow A_i$ via s_i for $1 \leq i \leq m$. We say S *takes A to B* if there is some S-derivation from A to B.

Now let γ be an arbitrary cost function which assigns to each edit operation $a \rightarrow b$ a nonnegative real number $\gamma(a \rightarrow b)$. Extend γ to a sequence of edit operations $S = s_1, s_2, \cdots, s_m$ by letting $\gamma(S) = \sum_{i=1}^{m} \gamma(s_i)$. (If $m = 0$, we define $\gamma(S) = 0$.) We now let the *edit distance* $\delta(A, B)$ from string A to string B be the minimum cost of all sequences of edit operations which transform A into B. Formally, $\delta(A, B) = \min\{\gamma(S) \mid S \text{ is an edit sequence taking } A \text{ to } B\}$.

We will assume henceforth that $\gamma(a \rightarrow b) = \delta(a, b)$ for all edit operations $a \rightarrow b$. (Equivalently, we may assume that $\gamma(a \rightarrow a) = 0$ and $\gamma(a \rightarrow b) + \gamma(b \rightarrow c) \geq \gamma(a \rightarrow c)$.) This leads to no loss of generality with respect to the class of distance functions we are considering, for if δ is the distance function associated with a cost function γ, it is easily verified that δ is also the distance function associated with the cost function γ' defined by $\gamma'(a \rightarrow b) = \delta(a, b)$, and γ' has the desired property.

Note that if δ were symmetric and strictly positive on each edit operation $a \rightarrow b$ for which $a \neq b$, then γ would be a metric on the space of all strings—hence our use of the term "distance." We remark also that cost functions which depend on the particular characters affected by an edit operation might be useful in spelling correction, where for example because of the conventional keyboard arrangement it may be far more likely that a character "A" be mistyped as an "S" than as a "Y."

3. *Traces*

To simplify our problem of finding the edit distance between two strings A and B, we define a cost function on some structures called *traces* and show that traces have the properties:

(P1) for every trace T from A to B, there is an edit sequence S taking A to B such that $\gamma(S) = \text{cost}(T)$;

(P2) for every edit sequence S taking A to B, there is a trace T from A to B such that $\text{cost}(T) \leq \gamma(S)$.

Thus, $\delta(A, B)$ is equal to the minimum cost trace from A to B, so we will be able to confine our attention to finding minimum cost traces.

Intuitively, a trace is a description of how an edit sequence S transforms A into B but ignoring the order in which things happen and any redundancy in S.

Consider the diagram:

String A: $x \quad y \quad z \quad w \quad t \quad w \quad x \quad z \quad x$

String B: $y \quad w \quad x \quad z \quad x \quad y \quad x \quad w$

A line in this diagram joining character position i of A to position j of B means that $B\langle j\rangle$ is derived from $A\langle i\rangle$, either directly if $A\langle i\rangle = B\langle j\rangle$ and S leaves $A\langle i\rangle$ unchanged or indirectly if S applies one or more change operations to $A\langle i\rangle$. Certain character positions of A are untouched by lines in our diagram; these positions represent characters of A deleted by S (either directly or perhaps as the result of one or more change operations followed by a delete). Similarly, certain positions of B are untouched by lines; these positions represent characters inserted into A by S.

Formally, a *trace from A to B* (or *trace* when the strings A and B are understood) is a triple (T, A, B), where T is any set of ordered pairs of integers (i, j) satisfying:

(1) $1 \leq i \leq |A|$ and $1 \leq j \leq |B|$;

(2) for any two distinct pairs (i_1, j_1) and (i_2, j_2) in T, (a) $i_1 \neq i_2$ and $j_1 \neq j_2$; (b) $i_1 < i_2$ iff $j_1 < j_2$.

A pair (i, j) describes a line joining position i of A to position j of B, and we say (i, j) *touches* those positions. Condition (1) ensures that our lines actually touch character positions of the respective strings. Condition (2a) ensures that each character position of either string is touched by at most one line; condition (2b) ensures that no two lines cross. Where there is no confusion, we will not distinguish between the triple (T, A, B) and the set of pairs T.

Let T be a trace from A to B. Let I and J be the sets of positions in A and B respectively not touched by any line in T. We define the cost of T:

$$\operatorname{cost}(T) = \sum_{(i,j)\in T} \gamma(A\langle i\rangle \to B\langle j\rangle) + \sum_{i\in I} \gamma(A\langle i\rangle \to \Lambda) + \sum_{j\in J} \gamma(\Lambda \to B\langle j\rangle).$$

Thus, the cost of T is just the cost of the edit sequence S taking A to B which consists of a change instruction $A\langle i\rangle \to B\langle j\rangle$ for each pair $(i, j) \in T$, a delete instruction $A\langle i\rangle \to \Lambda$ for every position i in A not touched by a line in T, and an insert instruction $\Lambda \to B\langle j\rangle$ for every position j in B not touched by a line in T. Hence, property (P1) of traces follows.

Traces may be composed. Let T_1 be a trace from A to B and let T_2 be a trace from B to C. It is readily verified that $T = T_1 \circ T_2$ is a trace from A to C, where $\circ$ denotes ordinary composition of relations.[1]

LEMMA 1. *$\operatorname{Cost}(T_1 \circ T_2) \leq \operatorname{cost}(T_1) + \operatorname{cost}(T_2)$, where T_1 is a trace from A to B and T_2 is a trace from B to C.*

The proof relies on our assumption that $\gamma(a \to b) = \delta(a, b)$ and is omitted.

To verify that property (P2) holds for traces, we show by induction on m that if $S = s_1, s_2, \cdots, s_m$ is a sequence of edit operations and $(A_0, A_1, \cdots, A_m)$ is an S-derivation (from A_0 to A_m), then there is a trace T from A_0 to A_m such that $\operatorname{cost}(T) \leq \gamma(S)$.

If $m = 0$, let $T = \{(i, i) \mid 1 \leq i \leq |A_0|\}$ be a trace from A_0 to A_0. Then $\operatorname{cost}(T) = 0 = \gamma(S)$ and the induction hypothesis holds.

If $m > 0$, by induction, there is a trace T_1 from A_0 to A_{m-1} such that $\operatorname{cost}(T_1) \leq \gamma(s_1, \cdots, s_{m-1})$. $A_{m-1} \Rightarrow A_m$ via $s_m = a \to b$, so there are strings σ and τ such that $A_{m-1} = \sigma a \tau$ and $A_m = \sigma b \tau$. Let T_2 be the trace from A_{m-1} to A_m defined by

$$T_2 = \{(i, i) \mid 1 \leq i \leq |\sigma|\} \cup \{(i, i + d) \mid |\sigma a| + 1 \leq i \leq |A_{m-1}|\} \cup L,$$

where $d = |b| - |a| \in \{-1, 0, 1\}$ and

$$L = \begin{cases} \{(|\sigma| + 1, |\sigma| + 1)\} & \text{if } s_m \text{ is a change instruction;} \\ \varnothing & \text{otherwise.} \end{cases}$$

Clearly, T_2 is a trace and $\operatorname{cost}(T_2) = \gamma(a \to b) = \gamma(s_m)$.

Now let $T = T_1 \circ T_2$. T is a trace from A_0 to A_m. By Lemma 1,

$$\operatorname{cost}(T) \leq \operatorname{cost}(T_1) + \operatorname{cost}(T_2) \leq \gamma(s_1, \cdots, s_{m-1}) + \gamma(s_m) = \gamma(S),$$

so property (P2) holds for S. By induction, it holds for all sequences S.

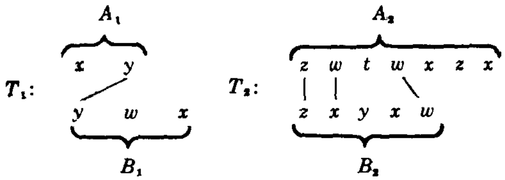
[1] $T_1 \circ T_2 = \{(i, j) \mid (i, k) \in T_1 \text{ and } (k, j) \in T_2 \text{ for some } k\}$.

From properties (P1) and (P2) of traces, we have:

THEOREM 1. $\delta(A, B) = \mathit{min}\{\mathit{cost}(T) \mid T \text{ is a trace from } A \text{ to } B\}$.

4. *Computation of Edit Distance*

Now return to the diagrammatic representation of a trace T from A to B. Let $A = A_1A_2$, $B = B_1B_2$, and suppose no line of T connects a character of A_i to a character of B_j for $i \neq j$, $i, j \in \{1, 2\}$. Then a trace (T, A, B) can be split into two traces (T_1, A_1, B_1) and (T_2, A_2, B_2) as illustrated.



Furthermore, $\mathrm{cost}(T) = \mathrm{cost}(T_1) + \mathrm{cost}(T_2)$, so if T is a least cost trace from A to B, then T_i is a least cost trace from A_i to B_i, $i \in \{1, 2\}$.

Every trace T from A to B can in fact be split into two traces T_1 and T_2 as above such that the lengths of A_2 and B_2 are each at most one but they are not both zero. This is the key idea for the following theorem, upon which the edit distance algorithm is based.

Notation. Let A and B be strings. Define $A(i) = A\langle 1 : i\rangle$, $B(j) = B\langle 1 : j\rangle$, and $D(i, j) = \delta(A(i), B(j))$, $0 \leq i \leq |A|$, $0 \leq j \leq |B|$. We note that by Theorem 1, $D(i, j)$ is also the cost of the least cost trace from $A(i)$ to $B(j)$.

THEOREM 2.

$$\begin{aligned} D(i, j) = \mathit{min}\{&D(i-1, j-1) + \gamma(A\langle i\rangle \to B\langle j\rangle), \\ &D(i-1, j) + \gamma(A\langle i\rangle \to \Lambda), \\ &D(i, j-1) + \gamma(\Lambda \to B\langle j\rangle)\} \end{aligned}$$

for all i, j, $1 \leq i \leq |A|$, $1 \leq j \leq |B|$.

PROOF. Let T be a least cost trace from $A(i)$ to $B(j)$. If $A\langle i\rangle$ and $B\langle j\rangle$ are both touched by lines in T, they must both be touched by the same line, since otherwise these lines in T would cross. Then at least one of the following three cases must hold:

Case 1. $A\langle i\rangle$ and $B\langle j\rangle$ are joined by a line of T (i.e. $(i, j) \in T$). Then the cost of T is $m_1 = D(i-1, j-1) + \gamma(A\langle i\rangle \to B\langle j\rangle)$, corresponding to the cost of transforming $A(i-1)$ to $B(j-1)$ plus the cost of changing $A\langle i\rangle$ to $B\langle j\rangle$.

Case 2. $A\langle i\rangle$ is not touched by any line in T. Then the cost of T is $m_2 = D(i-1, j) + \gamma(A\langle i\rangle \to \Lambda)$, corresponding to the costs of transforming $A(i-1)$ to $B(j)$ and deleting $A\langle i\rangle$.

Case 3. $B\langle j\rangle$ is not touched by any line in T. Then the cost of T is $m_3 = D(i, j-1) + \gamma(\Lambda \to B\langle j\rangle)$, corresponding to the costs of transforming $A(i)$ to $B(j-1)$ and inserting character $B\langle j\rangle$.

Since one of the three cases above must hold and $D(i, j)$ is to be a minimum, $D(i, j) = \min(m_1, m_2, m_3)$. □

THEOREM 3. $D(0, 0) = 0$; $D(i, 0) = \sum_{r=1}^{i} \gamma(A\langle r\rangle \to \Lambda)$; *and* $D(0, j) = \sum_{r=1}^{j} \gamma(\Lambda \to B\langle r\rangle)$, $1 \leq i \leq |A|$ *and* $1 \leq j \leq |B|$.

PROOF. The only (and hence least cost) trace from $A(i)$ to $B(j)$ when either i or $j = 0$ is $\varnothing$, and hence no lines touch $A(i)$ or $B(j)$. The theorem follows immediately from the definition of the cost of a trace. □

Theorems 2 and 3 justify that Algorithm X (below) correctly computes $D(i, j)$ for $0 \leq i \leq |A|$ and $0 \leq j \leq |B|$.

AGLORITHM X

1. $D[0, 0] := 0$;
2. **for** $i := 1$ **to** $|A|$ **do** $D[i, 0] := D[i-1, 0] + \gamma(A\langle i\rangle \to \Lambda)$;

```
for j := 1 to |B| do D[0, j] := D[0, j − 1] + γ(Λ → B⟨j⟩);
for i := 1 to |A| do
  for j := 1 to |B| do begin
    m1 := D[i − 1, j − 1] + γ(A⟨i⟩ → B⟨j⟩);
    m2 := D[i − 1, j] + γ(A⟨i⟩ → Λ);
    m3 := D[i, j − 1] + γ(Λ → B⟨j⟩);
    D[i, j] := min(m1, m2, m3);
    end;
```

By inspection, we see that the total amount of time used by Algorithm X is proportional to the number of assignment statements executed (exclusive of those implicit in the **for**-loops). This number is exactly $1 + |A| + |B| + 4 \times |A| \times |B|$, so the total time is $O(|A| \times |B|)$.

If an actual trace T from A to B of least cost is desired, Algorithm Y will print the pairs in T using only the information stored in array D by Algorithm X.

ALGORITHM Y

```
1. i := |A|; j := |B|;
2. while(i ≠ 0 & j ≠ 0) do
3.   if D[i, j] = D[i − 1, j] + γ(A⟨i⟩ → Λ) then i := i − 1;
4.   else if D[i, j] = D[i, j − 1] + γ(Λ → B⟨j⟩) then j := j − 1;
5.   else begin
6.     print((i, j));
7.     i := i − 1; j := j − 1;
8.     end;
```

In order to prove that Algorithm Y works correctly, we consider for every pair of natural numbers I and J the behavior of the algorithm when started at step 2 with variables i and j initialized to I and J respectively. Let $T(I, J)$ be the set of pairs printed by the algorithm if the execution eventually terminates, and $T(I, J)$ is undefined otherwise.

THEOREM 4. *If* $0 \le I \le |A|$ *and* $0 \le J \le |B|$, *then* $T(I, J)$ *is defined*, $\mathbf{T} = (T(I, J), A(I), B(J))$ *is a trace, and cost* $(\mathbf{T}) = D(I, J)$.

PROOF. We proceed by induction on the sum $I + J$.

The theorem is vacuously true for $I + J < 0$.

Now let $r \ge 0$ and suppose the theorem holds for all I', J' such that $I' + J' < r$. Let $I + J = r$. If either I or J is 0, step 2 terminates immediately and $T(I, J) = \varnothing$ is the only trace from $A(I)$ to $B(J)$; hence its cost is minimal. If neither I nor J is zero, we have three cases:

Case 1. The test in step 3 succeeds. Then $D(I, J) = D(I - 1, J) + \gamma(A\langle I\rangle \to \Lambda)$. The algorithm then proceeds by decrementing i and returning to step 2. Variable i now has the value $I - 1$, and j is unchanged. By induction, $T(I - 1, J)$ is defined, and $\mathbf{T} = (T(I - 1, J), A(I - 1), B(J))$ is a trace of cost $D(I - 1, J)$. No output was produced before returning to step 2,.so $T(I, J) = T(I - 1, J)$, and $\mathbf{T}' = (T(I, J), A(I), B(J))$ is a trace. Then

$$\text{cost}(\mathbf{T}') = \text{cost}(\mathbf{T}) + \gamma(A\langle I\rangle \to \Lambda) = D(I - 1, J) + \gamma(A\langle I\rangle \to \Lambda) = D(I, J).$$

Case 2. The test in step 3 fails but the one in step 4 succeeds. The proof for this case is exactly analogous to case 1.

Case 3. The tests in steps 3 and 4 both fail. Hence $D(I, J) \ne D(I - 1, J) + \gamma(A\langle I\rangle \to \Lambda)$ and $D(I, J) \ne D(I, J - 1) + \gamma(\Lambda \to B\langle J\rangle)$. By Theorem 2, it must be the case that $D(I, J) = D(I - 1, J - 1) + \gamma(A\langle I\rangle \to B\langle J\rangle)$.

The block from steps 5–8 is then executed. This causes the pair (I, J) to be printed, and when step 2 is reentered, both i and j have been decremented. By induction, $T(I - 1, J - 1)$ is defined, and $\mathbf{T} = (T(I - 1, J - 1), A(I - 1), B(I - 1))$ is a trace of cost $D(I - 1, J - 1)$. Hence, $T(I, J) = \{(I, J)\} \cup T(I - 1, J - 1)$, and $\mathbf{T}' = (T(I, J),$

$A(I), B(J))$ is a trace. Then

$$\text{cost}(\mathbf{T}') = \text{cost}(\mathbf{T}) + \gamma(A\langle I\rangle \to B\langle J\rangle) = D(I-1, J-1) + \gamma(A\langle I\rangle \to B\langle J\rangle) = D(I, J).$$

Hence, in all three cases, the theorem holds for I and J. By induction, the theorem holds for all I and J. $\square$

Algorithm Y when started at the beginning first enters step 2 with $i = |A|$ and $j = |B|$. By Theorem 4, it eventually terminates and prints the pairs in $T(|A|, |B|)$, which is a least cost trace from A to B as desired.

We note that in all three cases of the proof of Theorem 4, either i or j (or both) is decremented, and Algorithm Y terminates when either reaches 0. Hence, the loop is executed at most $|A| + |B|$ times, so the total running time of Algorithm Y is $O(|A| + |B|)$.

5. *Longest Common Subsequences*

Let U and V be strings. U is a *subsequence* of V if there exist integers $1 \leq r_1 < r_2 < \cdots < r_n \leq |V|$ such that $U\langle i\rangle = V\langle r_i\rangle$, $1 \leq i \leq n = |U|$. Given two strings A and B, U is a *common subsequence* of A and B if U is a subsequence of both A and B.

Let $\rho(A, B)$ be the length of the longest common subsequence of A and B. It is immediate from the definition of a trace that $\rho(A, B)$ is also the maximum number of pairs (i, j) in any trace from A to B for which $A\langle i\rangle = B\langle j\rangle$. Let T be such a trace.

Define γ so that the cost of an insert or a delete operation is 1, and let the cost of a change operation $a \to b$ be 0 if $a = b$ and 2 if $a \neq b$. Under this cost assignment, $\text{cost}(T) = |A| + |B| - 2\rho(A, B)$. T is a least cost trace from A to B, so $\text{cost}(T) = \delta(A, B)$. Hence, $\rho(A, B) = (|A| + |B| - \delta(A, B))/2$ can be computed in time $O(|A| \times |B|)$ using Algorithm X. The longest common subsequence itself can be found easily from T which in turn can be obtained using Algorithm Y.

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Format. Manuscripts should be submitted in triplicate (the original on bond-weight paper) under cover of a submittal letter signed by the author. The text should be double spaced *on one side* of the paper. Typed manuscripts are preferred, but good reproductions of internal reports are acceptable (if text runs on both sides of pages, submit four copies). Authors' names should be given without titles or degrees. The name and address of the organization for which the work was carried out should be given. If the paper has previously been presented at a technical meeting, this fact, giving the date and sponsoring society, should appear in a footnote on the first page. Acknowledgments of funding sources should also be given in a footnote on the first page.

The usefulness of articles published in ACM periodicals is greatly enhanced when each paper includes information which insures proper indexing, classification, retrieval, and dissemination. To this effect authors should include in the manuscript:

(a) descriptive title;
(b) author names—with addresses in a footnote;
(c) informative abstract;
(d) content indicators of two types:
  (i) appropriate key words and key phrases,
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The following suggestions may be useful in preparing this information.

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