Graham (1982): Galil, Haber and Yung (1989); Goldberg, Goldberg and Paterson (2001); Goldstein and Waterman (1987); Grove (1995); Hochbaum (1997); Holyer (1981); Kannan and Warnow (1994); Kearney, Hayward and Meijer (1997); Lathrop (1994); Lent and Mahmoud (1996); Lipton (1995); Lyngso and Pedersen (2000); Ma, Li and Zhang (2000); Maier and Storer (1977); Pe'er and Shamir (1998); Pierce and Winfree (2002); Storer (1977); Thomassen (1997); Unger and Moult (1993) and Wareham (1995).

Exercises

8.1 Determine whether the following statements are correct or not

(1) If a problem is NP-complete, then it cannot be solved by any polynomial algorithm in worst cases.

(2) If a problem is NP-complete, then we have not found any polynomial algorithm to solve it in worst cases.

(3) If a problem is NP-complete, then it is unlikely that a polynomial algorithm can be found in the future to solve it in worst cases.

(4) If a problem is NP-complete, then it is unlikely that we can find a polynomial algorithm to solve it in average cases.

(5) If we can prove that the lower bound of an NP-complete problem is exponential, then we have proved that NP $\neq$ P.

8.2 Determine the satisfiability of each of the following sets of clauses.

(1)

$$
\begin{array}{ccccc}
-X_1 & \vee & -X_2 & \vee & X_3 \\
X_1 & & & & \\
X_2 & \vee & X_3 & & \\
-X_3 & & & &
\end{array}
$$

(2)

$$
\begin{array}{ccccc}
X_1 & \vee & X_2 & \vee & X_3 \\
-X_1 & \vee & X_2 & \vee & X_3 \\
X_1 & \vee & -X_2 & \vee & X_3 \\
X_1 & \vee & X_2 & \vee & -X_3 \\
-X_1 & \vee & -X_2 & \vee & X_3
\end{array}
$$

$$
\begin{array}{llllll}
 & X_1 & \vee & -X_2 & \vee & -X_3 \\
 & -X_1 & \vee & X_2 & \vee & -X_3 \\
 & -X_1 & \vee & -X_2 & \vee & -X_3 \\
(3) & -X_1 & \vee & X_2 & \vee & X_3 \\
 & X_1 & \vee & X_2 & & \\
 & X_3 & & & & \\
(4) & X_1 & \vee & X_2 & \vee & X_3 \\
 & X_1 & & & & \\
 & X_2 & & & & \\
(5) & -X_1 & \vee & X_2 & & \\
 & -X_2 & \vee & X_3 & & \\
 & -X_3 & & & &
\end{array}
$$

8.3 We all know how to prove that a problem is NP-complete. How can we prove that a problem is not NP-complete?

8.4 Complete the proof of the NP-completeness of the exact cover problem as described in this chapter.

8.5 Complete the NP-completeness of the sum of subset problem as described in this chapter.

8.6 Consider the following problem. Given two input variables a and b, return "**YES**" if $a > b$ and "**NO**" if otherwise. Design a non-deterministic polynomial algorithm to solve this problem. Transform it into a Boolean formula such that the algorithm returns "**YES**" if and only if the transformed Boolean formula is satisfiable.

8.7 Maximal clique decision problem: A maximal clique is a maximal complete subgraph of a graph. The size of a maximal clique is the number of vertices in it. The clique decision problem is to determine whether there is a maximal clique at least size k for some k in a graph or not. Show that the maximal clique decision problem is NP-complete by reducing the satisfiability problem to it.

8.8 Vertex cover decision problem: A set S of vertices of a graph is a vertex cover of this graph if and only if all edges of the graph are incident to at least one vertex in S. The vertex cover decision problem is to determine whether a graph has a vertex cover having at worst k vertices. Show that the vertex cover decision problem is NP-complete.

8.9 Traveling salesperson decision problem: Show that the traveling salesperson decision problem is NP-complete by proving that the Hamiltonian cycle decision problem reduces polynomially to it. The definition of Hamiltonian cycle decision problem can be found in almost any textbook on algorithms.

8.10 Independent set decision problem: Given a graph G and an integer k, the independent set decision problem is to determine whether there exists a set S of k vertices such that no two vertices in S are connected by an edge. Show that the independent set problem is NP-complete.

8.11 Bottleneck traveling salesperson decision problem: Given a graph and a number M, the bottleneck traveling salesperson decision problem is to determine whether there exists a Hamiltonian cycle in this graph such that the longest edge of this cycle is less than M. Show that the bottleneck traveling salesperson decision problem is NP-complete.

8.12 Show that 3-coloring $\propto$ 4-coloring $\propto$ k-coloring.

8.13 Clause-monotone satisfiability problem: A formula is monotone if each clause of it contains either only positive variables or only negative variables. For instance

$$F = (X_1 \vee X_2) \,\&\, (-X_3) \,\&\, (-X_2 \vee -X_3)$$

is a monotone formula. Show that the problem of deciding whether a monotone formula is satisfiable or not is NP-complete.

8.14 Read Theorem 15.7 of Papadimitriou and Steiglitz (1982) for the NP-completeness of the 3-dimensional matching problem.