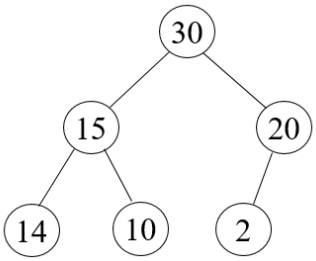
Department of Computer Science and Engineering
National Sun Yat-sen University
Data Structures - Final Exam., Jan. 6, 2020

1. Suppose that the division method (mod 11) is used in the *hashing* function. The *linear probing* method is applied when a collision occurs. Please give the hash table after all numbers have been inserted into the table according to the input order: 18, 22, 19, 12, 16, 33, 2, 7. (10%)

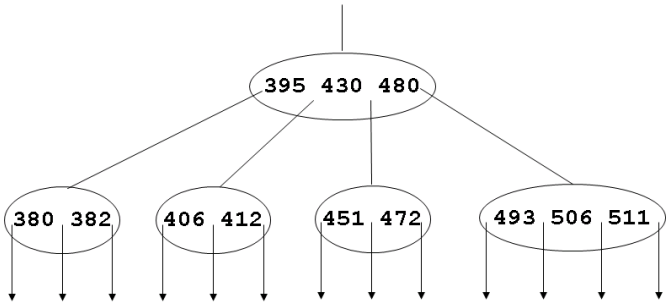
2. Suppose that we have a maximum *heap* (maximum is stored in the root) as shown in the following figure.

   (a) Please draw the figure after 25 and 35 are inserted. (6%)

   (b) Please draw the figure after the largest and the second largest are deleted from the heap (25 and 35 are not inserted). (6%)



3. Please draw the tree after 485 and 496 are inserted into the following B tree of order 5. (6%)



4. Starting with an empty *red-black tree*, suppose the key insertion sequence is 15, 12, 7, 35, 45, 80, 5. Draw the red-black tree after each insertion, and indicate the node colors (R for red, and B for black). (12%).

5. Suppose that we are given n identifiers $a_1, a_2, \ldots, a_n$ with $a_1 < a_2 < \ldots < a_n$, and the probability p_i of successful search for a_i. Besides, the probability of each unsuccessful search is zero. The *optimal binary search tree* (OBST) is the tree with the minimal search cost by considering p_i. Let C_{ij} denote the cost of the OBST constructing from $a_i, a_{i+1}, \ldots, a_j$, where $1 \le i \le j \le n$. Please present the *dynamic programming* method (recursive formula) for solving C_{ij}. (10%)

6. Given a permutation $p_1p_2\ldots p_n$ of $123\ldots n$, a *homing operation* (going home at once), denoted as h_j, is to put one number p_i into its home position j and to shift

the numbers between positions i and j, where the home position of p_i is j if the value of p_i is j. Note that the leftmost position is position 1 and the rightmost position is position n. For example, after h_2 is applied, 81374256 will become 82137456, and after h_7 is applied, 81374256 will become 81342576. For each of the following questions, please write down the permutation following each used homing operation

(a) How do you sort 81374256 into 12345678 with the minimum number of homing operations? (5%)

(b) How do you sort 67145823 into 12345678 with the minimum number of homing operations? (5%)

7. Explain each of the following terms. (16%)
   (a) stable sorting
   (b) external sorting
   (c) AVL tree
   (d) splay tree

8. Write a recursive C/C++ function to count the number of even numbers and number of odd numbers in a *binary tree*, where each node stores one number (not a binary search tree). (12%)

```
class TreeNode {
   int data;      // the number stored in the node
   TreeNode *leftChild, *rightChild;
};
int Count(….) or void Count (…)
{
```

Please write the body of the function.

```
} // end of Count( )
```

9. Please write a recursive C/C++ function to perform the *recursive merge sort*. To implement your merge sort, you can call the following 2-*way merge* function as a basic function, which merges two sorted arrays into a single one. In other words, you need not write the body of the 2-way merge function. (12%)

```
void twoway(int a[ ], int b[ ], int c[ ], int na, int nb)
// a[ ] and b[ ] are input sorted arrays
// c[ ] is the output sorted array after a[ ] and b[ ] are merged
// na and nb are the lengths of a[ ] and b[ ], respectively
//You can call twoway(…) directly.
```

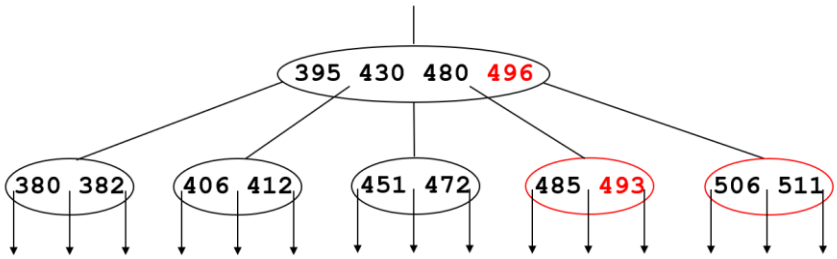
Answer:

1. hash table

0	1	2	3	4	5	6	7	8	9	10

2. heap

3. B tree



4. red-black tree

6. Homing operation

(a) h_8, h_7, h_2 or h_8, h_2, h_7

(b) h_8, h_7, h_6, h_5, h_4 or h_1, h_2, h_3, h_4, h_5 or h_8, h_7, h_6, h_2, h_3